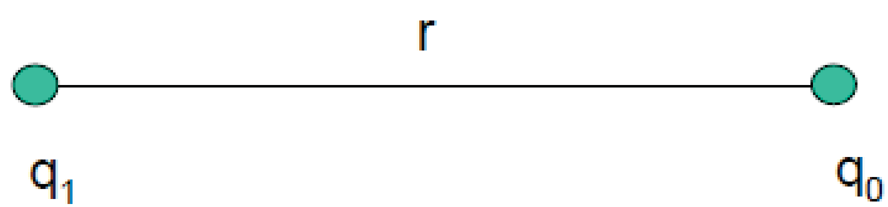
Lecture (03) Electric Charges and Electric Field (3)

By:

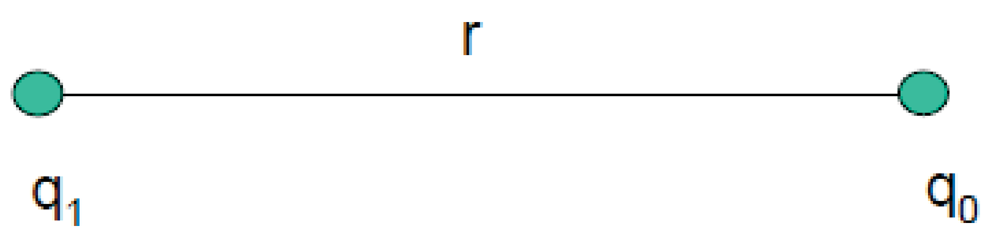
Dr. Ahmed ElShafee

Electric Field - Discovery

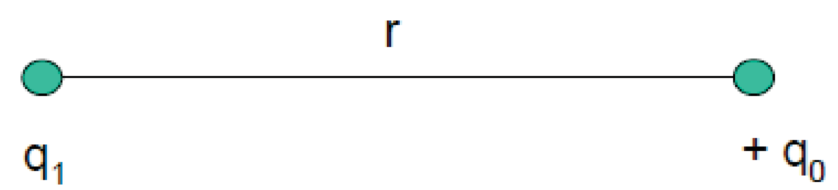
- Whenever charges are present and if I bring up another charge, it will feel a net Coulomb force from all the others.
- It is convenient to say that there is a field there equal to the force per unit positive charge. $\mathbf{E}=\mathbf{F}/q_0$.
- The question is how does charge q0 know about charge q1 if it does not “touch it”? Even in a vacuum!
- We say there is a field produced by q1 that extends out in space everywhere.



- If another charged object enters a region where an electrical field is present, it will be subject to an electrical force.
- The direction of the electric field is along r and points in the direction a positive test charge would move.
- This idea was proposed by Michael Faraday in the 1830's.

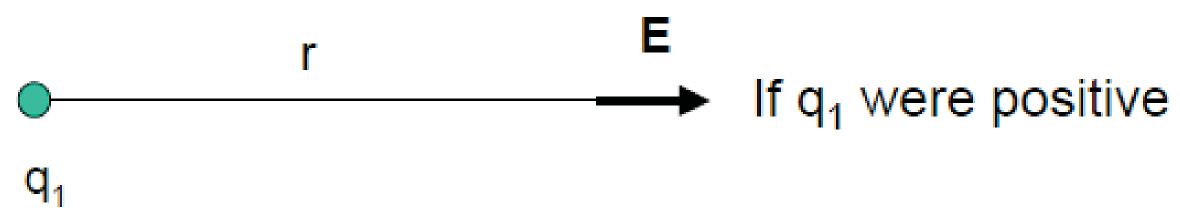


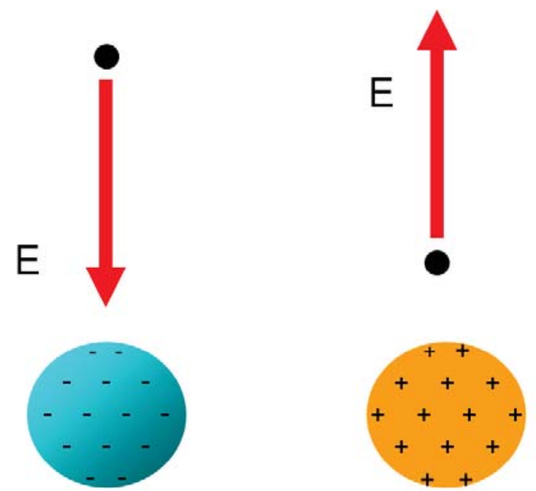
Electric Field – Quantitative Definition



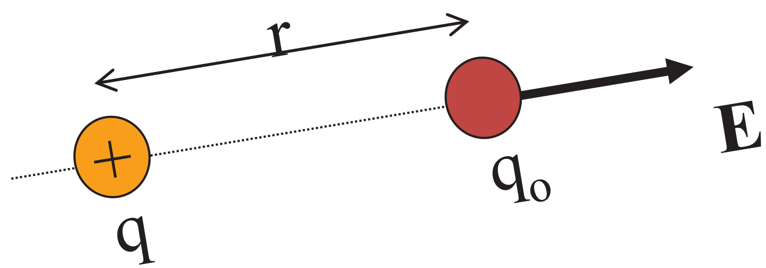
- The Coulomb force is $\mathbf{F} = k\, q_1 q_0/r^2$
- The force per unit charge is $\mathbf{E} = \mathbf{F}/q0$
- Then the electric field at r is $\mathbf{E} = k\, q_1/r^2$ due to the point charge q1 .
- The units are Newton/Coulomb.

- The electric field has direction and is a vector.
- The direction is the direction a unit positive test charge would move.
- This is called a field line.

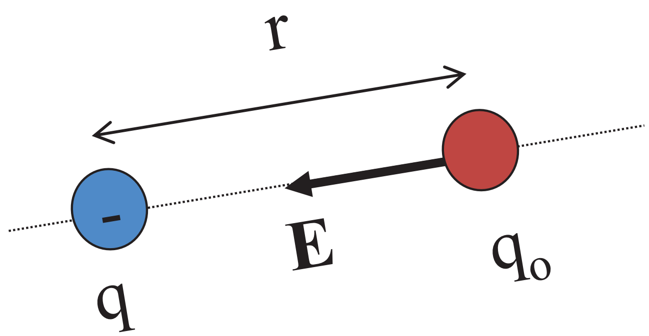




- If $q>0$, field at a given point is ***radially outward*** from q.



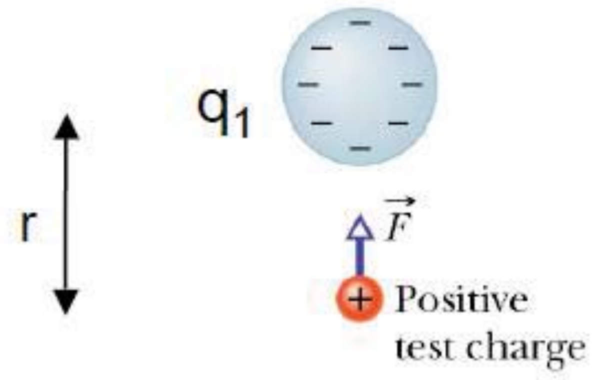
- If $q<0$, field at a given point is ***radially inward*** from q.

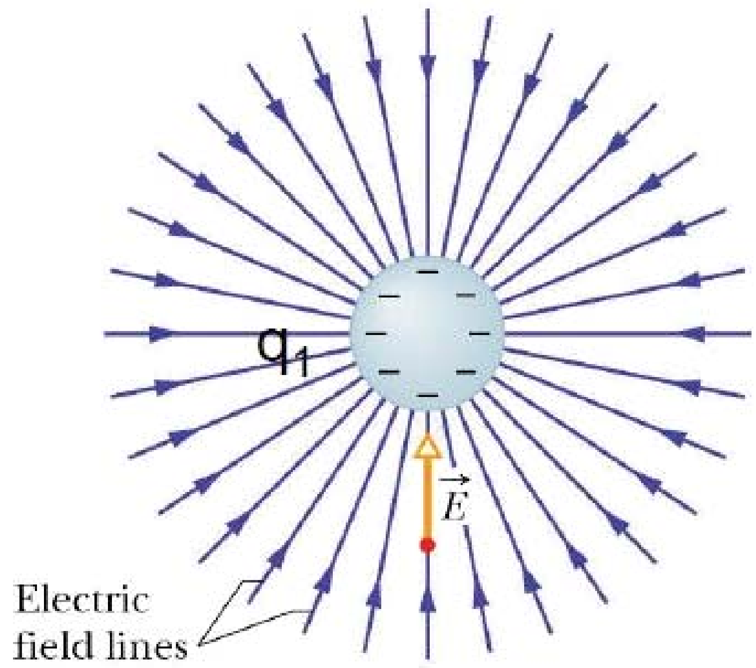


- Example of field lines for a point negative charge.
- Place a unit positive test charge at every point r and draw the direction that it would move
- The blue lines are the field lines.
- The magnitude of the electric field is

$$\vec{E} = \frac{kq_1}{r^2}\,\hat{r}$$

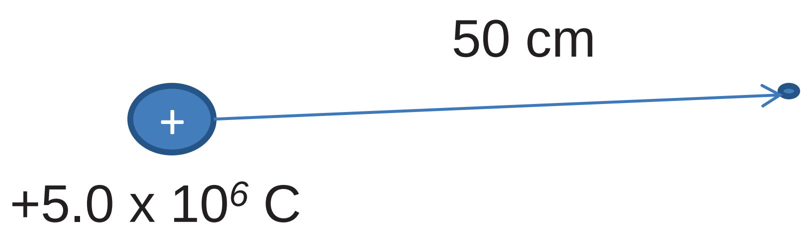
- The direction of the field is given by the line itself





Example 01

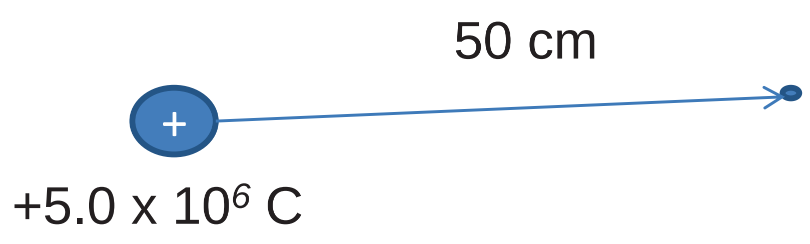
- Calculate the magnitude and direction of the electric field at a point P which is 50 cm to the right of a point charge $Q = +5.0 \times 10^{-6}$ C.



$E = K \cdot q / r^2$

$= 8.99 \times 10^9 \times 5.0 \times 10^{-6} / (0.5)^2 =$

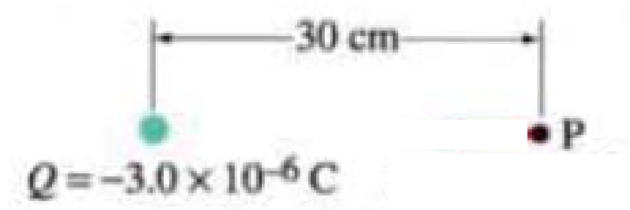
1.8×10^5 N/C



Direction .. As Q is positive charge, field is going outward Q

Example 02

- Calculate the magnitude and direction of the electric field at a point P which is 30cm to the right of a point charge $Q = - 3.0 \times 10^{-6}$ C.



$E = K . q / r^2$

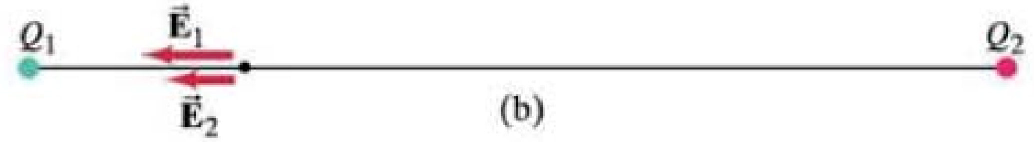
$= 8.99 \times 10^9 \times 3.0 \times 10^{-6} / (0.3)^2 =$

2.9967×10^5 N/C

Direction .. As Q is negative charge, field is going toward Q

Superposition

- If the electric field at a given point in space is due to more than one charge, the individual fields (call them E_1, E_2 , etc.) due to each charge are added vectorially to get the total field at that point:
- $\vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots$.

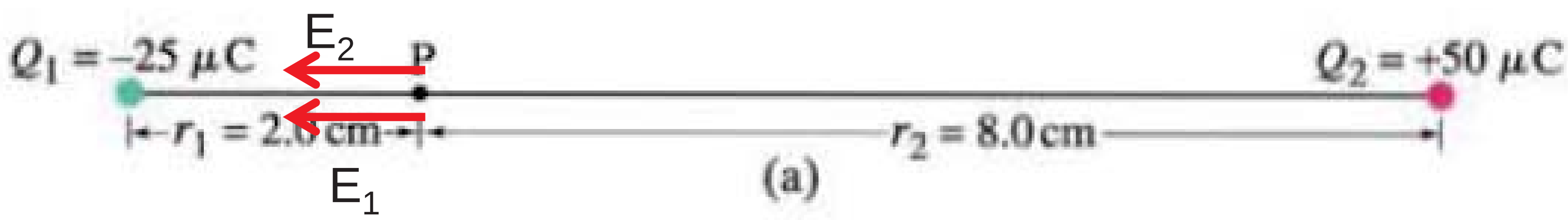


(b)

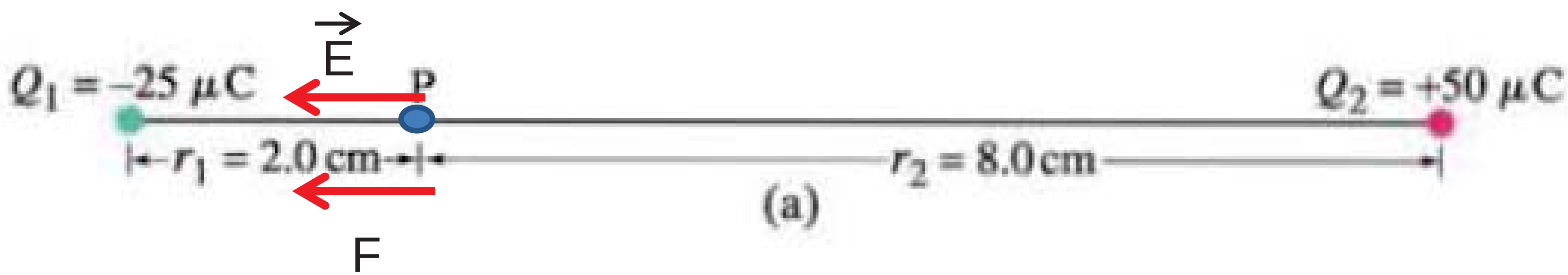
Example 03

- Two point charges are separated by a distance of 10 cm.
- One has a charge of *–25uC* and the other +50 *uC.*
- (a) Determine the direction and magnitude of the electric field at a point P between the two charges that is 2.0cm from the negative charge
- *(b)* If an electron (mass = 9.11 *x* 10^{31} kg) is placed at rest at P and then released, what will be its initial acceleration (direction and magnitude)?

(a)



- $\vec{E1} = K \times Q_1 / (r_1)^2 = 8.99 \times 10^9 \times 25 \times 10^{-6} / (.02)^2 = 5.6 \times 10^8$ N/C
- $\vec{E2} = 8.99 \times 10^9 \times 50 \times 10^{-6} / (.08)^2 = 0.702 \times 10^8$ N/C
- $\vec{E} = 6.3 \times 10^8$ N/C

- 

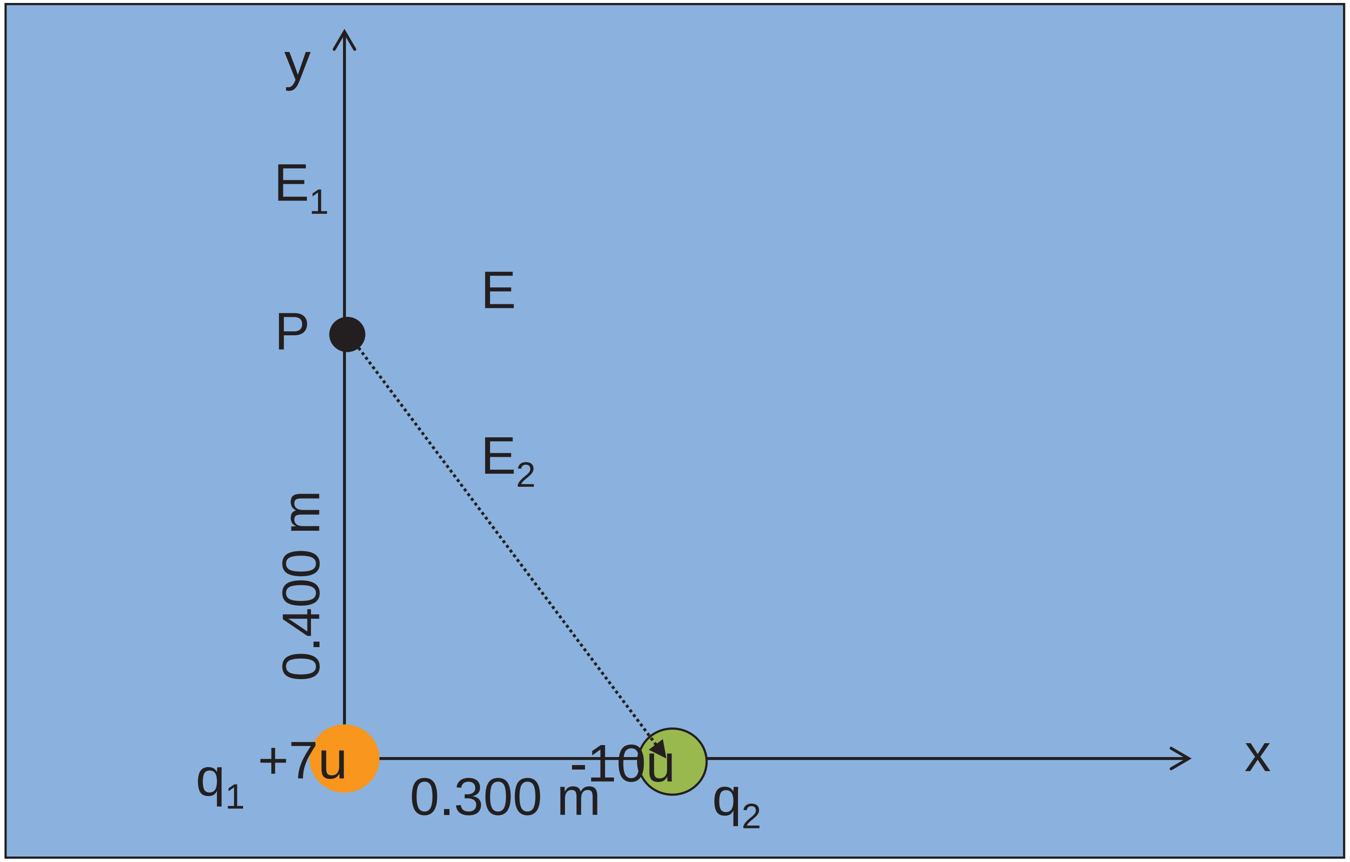
- $F = E \times q = 1.6 \times 10^{-19} \times 6.3 \times 10^{8} = 1.008 \times 10^{-10}$ N
- $a = F/m = 1.008 \times 10^{-10} / 9.11 \times 10^{-31} = 1.12 \times 10^{20}$ m/s^2

Problem-Solving Strategy

- Electric Forces and Fields
  - **Units:**
  - For calculations that use the Coulomb constant, k_e, charges must be in coulombs, and distances in meters.
  - Conversion are required if quantities are provided in other units.
  - **Applying Coulomb's law to point charges.**
  - It is important to use the superposition principle properly.
  - Determine the individual forces first.
  - Determine the vector sum.
  - Determine the magnitude and/or the direction as needed.

Example 04:

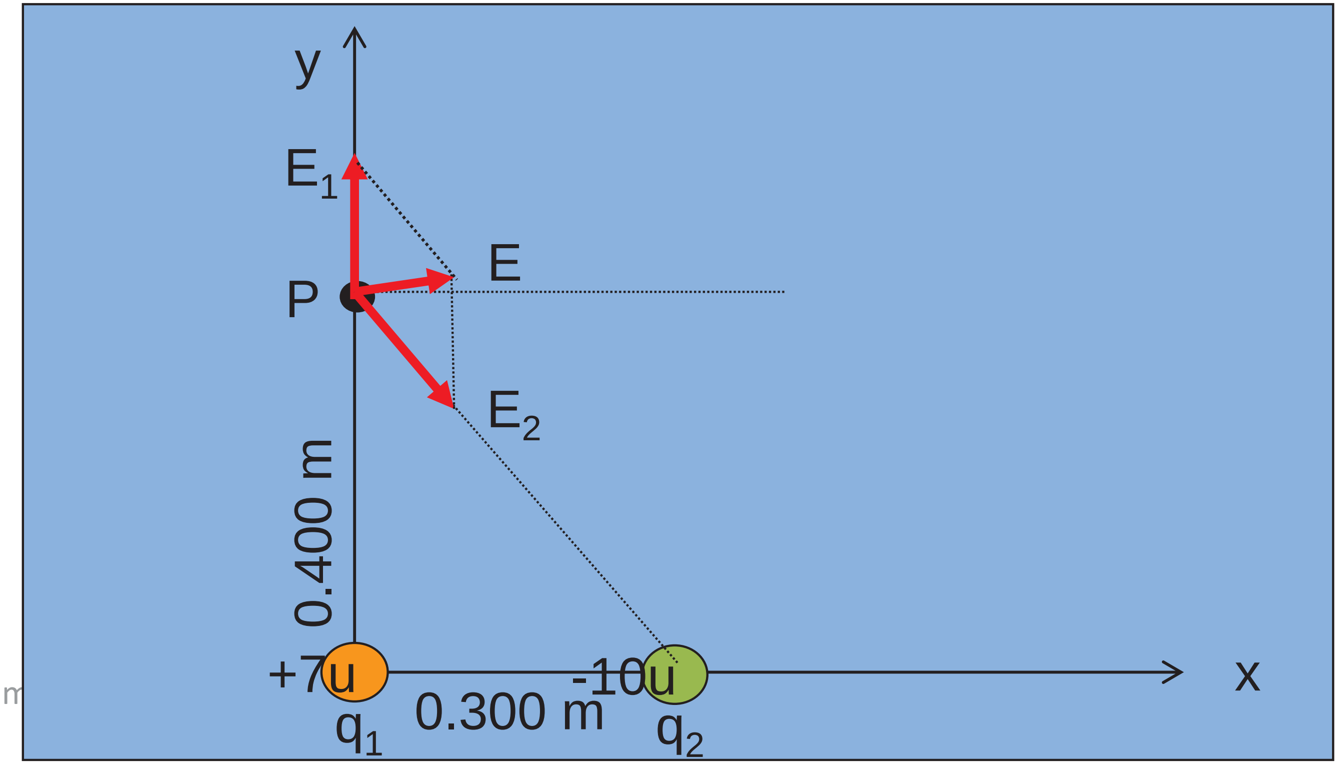
Charge q_1=7.00 μC is at the origin, and charge q_2=-10.00 μC is on the x axis, 0.300 m from the origin. Find the electric field at point P, which has coordinates (0,0.400) m.



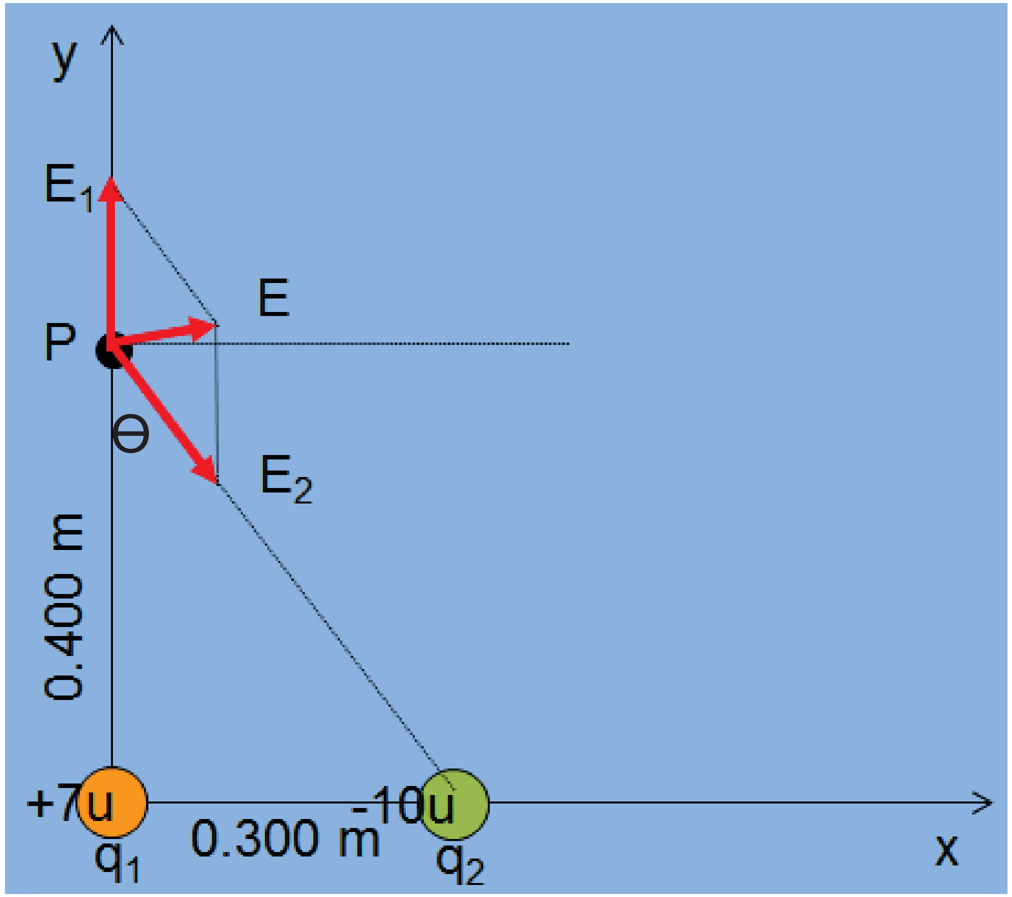
Solution strategy:

- First find the field at point P due to charge q_1 and q_2.
- Field E_1 at P due to q_1 is vertically upward.
- Field E_2 at due to q_2 is directed towards q_2.
- The net field at point P is the vector sum of E_1 and E_2.
- The magnitude is obtained with

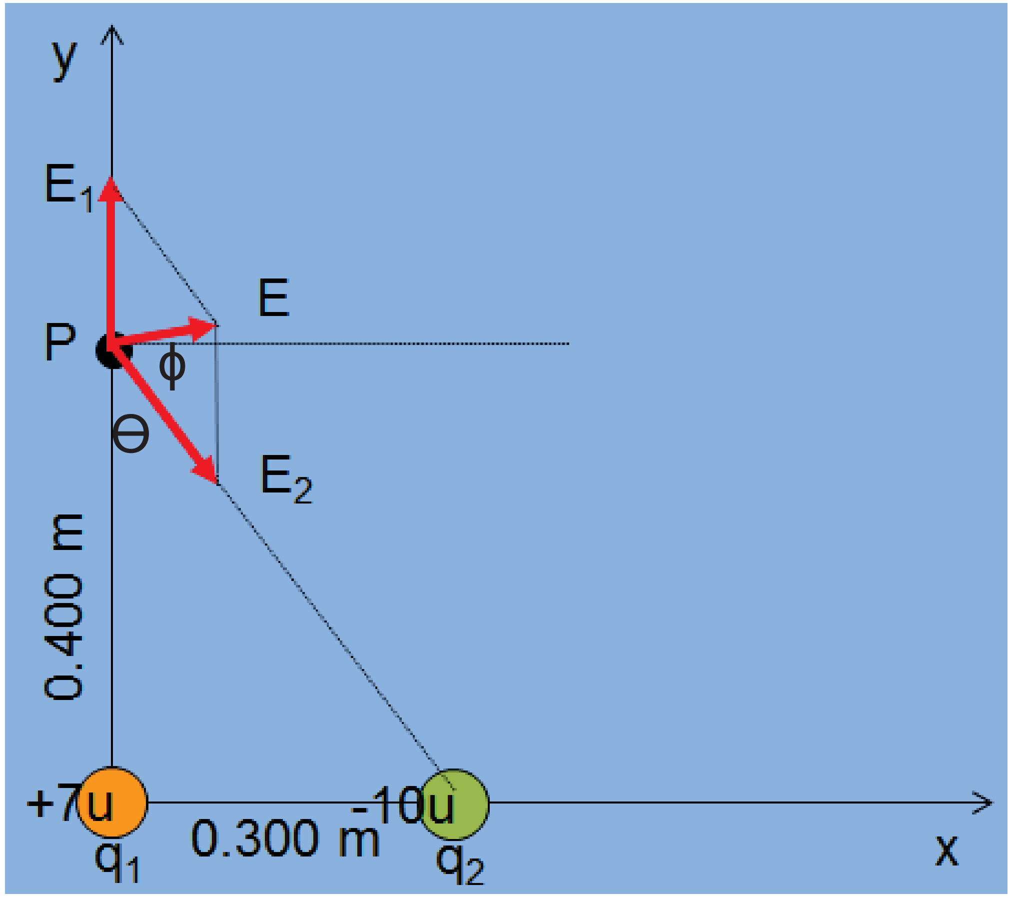
$$E = k_e \frac{|q|}{r^2}$$



- $E_1 = k \times q_1 / r_1^2 = 8.99 \times 10^9 \times 7 \times 10^{-6} \times (1/0.4^2) = 3.9 \times 10^5$ N/C
- $R_2 = (0.3^2 + 0.4^2)^{0.5} = 0.5$ m
- $E_2 = k \times q_2 / r_2^2 = 8.99 \times 109 \times 10 \times 10\text{-}6 \times (1/0.5^2) = 3.6 \times 10^5$ N/C
- $E_{px} = 3.6 \times 10^5 \times \sin \Theta = 3.6 \times 10^5 \times (.3/.5) = 2.1 \times 10^5$ N/C
- $E_{py} = 3.9 \times 10^5 - 3.6 \times 10^5 \times \cos \Theta = 3.9 \times 10^5 - (3.6 \times 10^5 \times 4/5) = 1.02 \times 10^5$ N/C

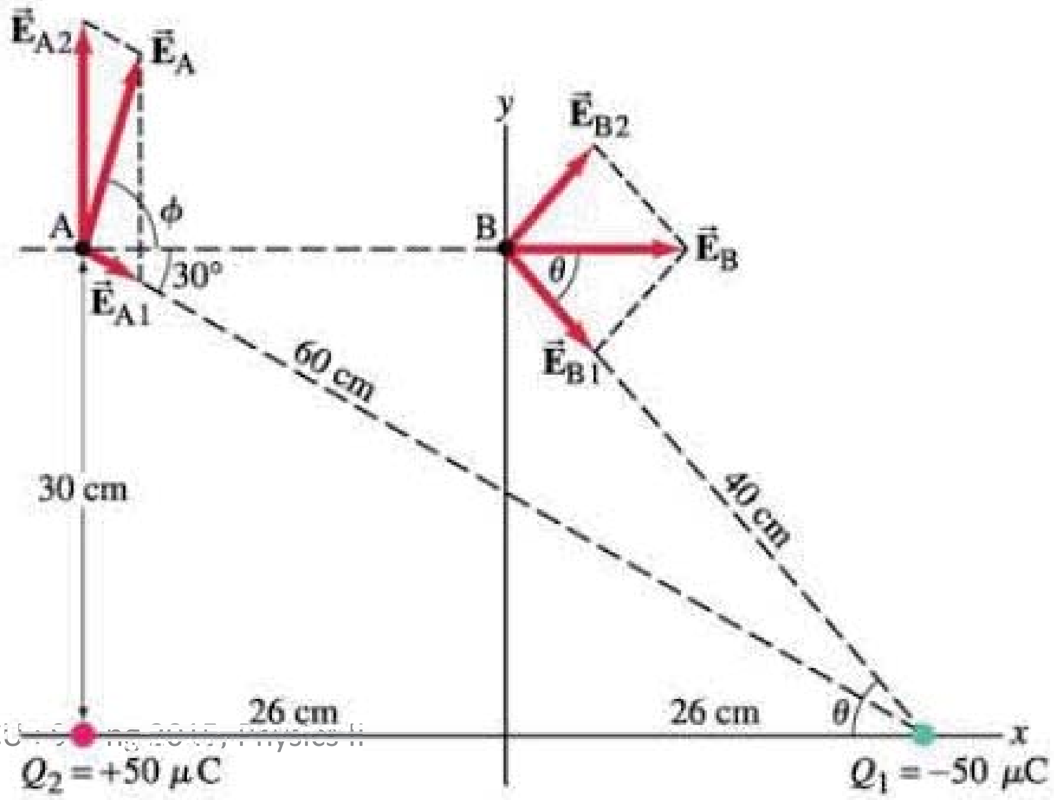


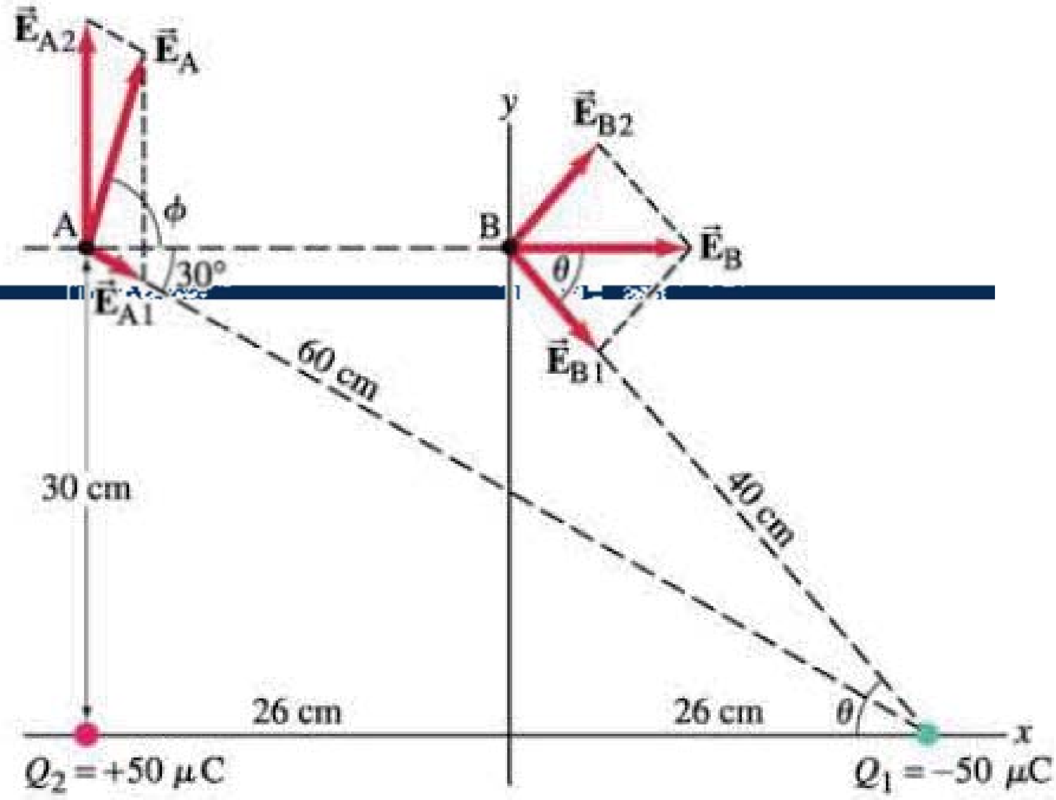
- $E_p = (E_{xp}{}^2 + E_{yp}{}^2)^{0.5} = ((2.1 \times 10^5)^2 + (1.02 \times 10^5)^2)^{0.5} = 2.33 \times 10^5$ N/C
- $\phi = \tan^{-1}(1.02 \times 10^5 / 2.1 \times 10^5) = \tan^{-1}(0.4857) = 25.9°$



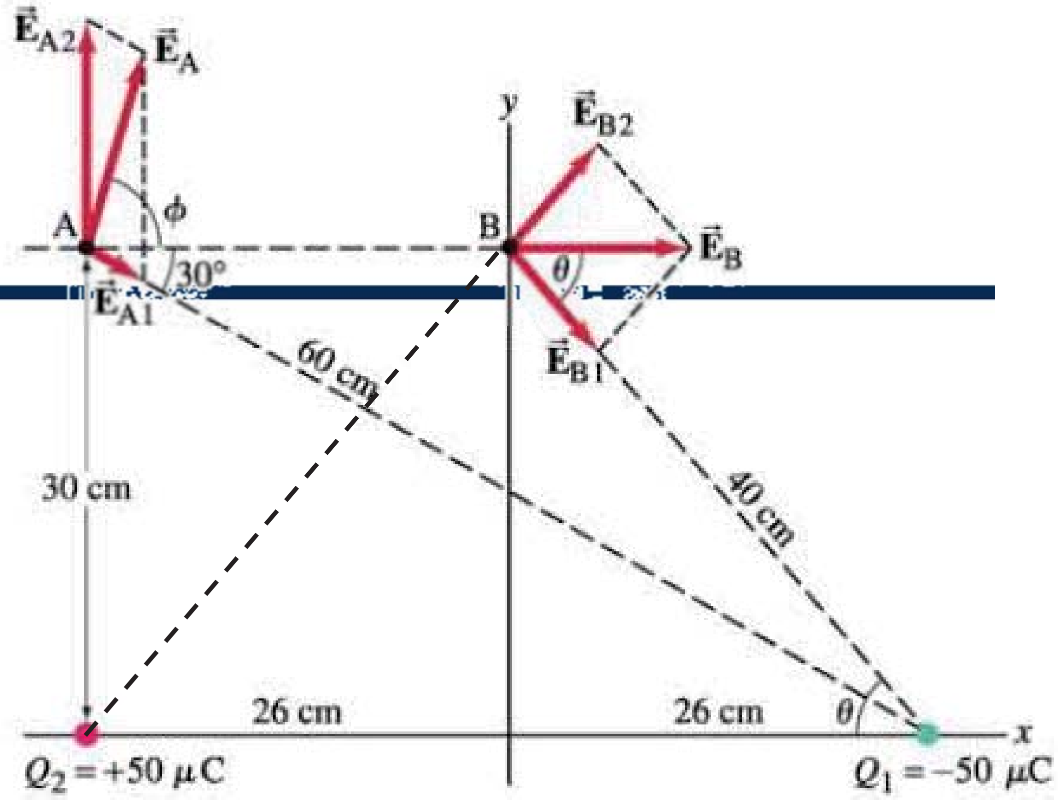
Example 05

- Calculate the total electric field
- (a) at point A and
- (b) at point B
- due to both charges, Q1 and *Q2*



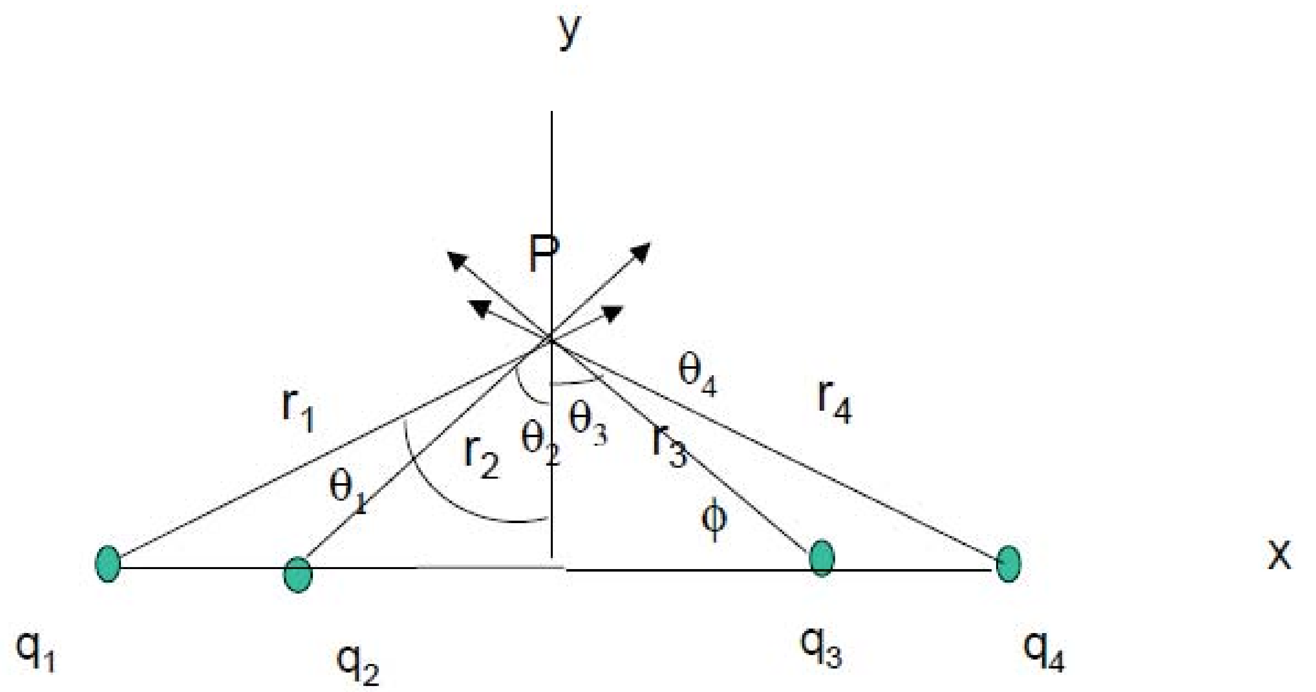


- $E_{A1} = 8.99 \times 10^9 \times 50 \times 10^{-6} / (0.6)^2 = 1.25 \times 10^6$ N/C
- $E_{A2} = 8.99 \times 10^9 \times 50 \times 10^{-6} / (0.3)^2 = 4.99 \times 10^6$ N/C
- $E_{AX} = 1.25 \times 10^6 \times \cos 30 = 1.1 \times 10^6$ N/C
- $E_{AY} = (-1.25 \times 10^6 \times \sin 30) + 4.99 \times 10^6 = 4.37 \times 10^6$ N/C
- $EA = ((1.1 \times 10^6)^2 + (4.37 \times 10^6)^2)^{0.5} = 4.5 \times 10^6$ N/C
- $\phi = \tan^{-1}(4.37 \times 10^6 / 1.1 \times 10^6) = 75.87°$



- $E_{B1} = 8.99 \times 10^9 \times 50 \times 10^{-6} / (0.4)^2 = 2.8 \times 10^6$ N/C
- $E_{B2} = 8.99 \times 10^9 \times 50 \times 10^{-6} / (0.4)^2 = 2.8 \times 10^6$ N/C
- $E_{BX} = 2 \times 2.8 \times 10^6 \times \cos \Theta = 2 \times 2.8 \times 10^6 \times 26/40 = 3.64 \times 10^6$ N/C
- $E_{BY} = 0$ N/C
- $E_B = 3.64 \times 10^6$ N/C
- Angle = 0°

- 4 equal charges symmetrically spaced along a line. What is the field at point P? (y and x = 0)



$$E_y = k\sum_{i=1}^{4} q_i \cos\theta_i / r_i^2 \qquad E_y = k\sum_{i=1}^{\infty} \Delta q_i \cos\theta_i / r_i^2$$

MCQ

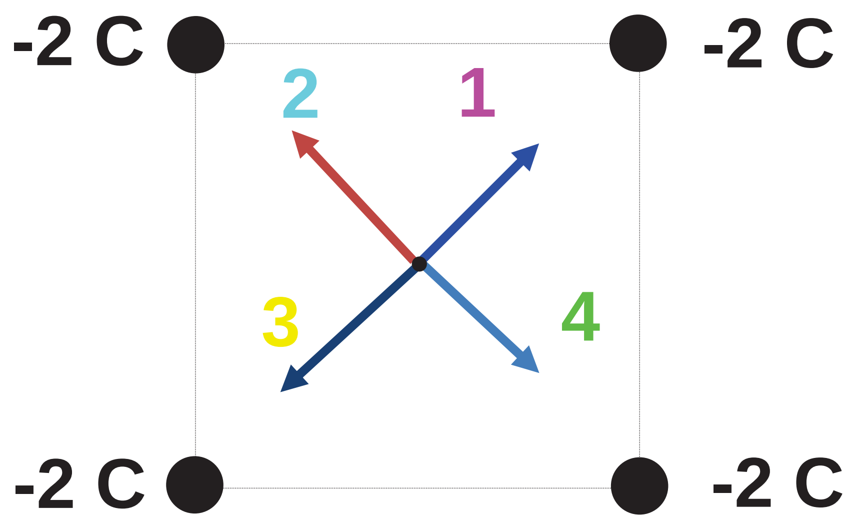
1. What is the electric field at the center of the square?

a. $4 \times K \times 2 / r^2$

b. $-4 \times K \times 2 / r^2$

c. Zero

d. $K \times 2 / r^2$



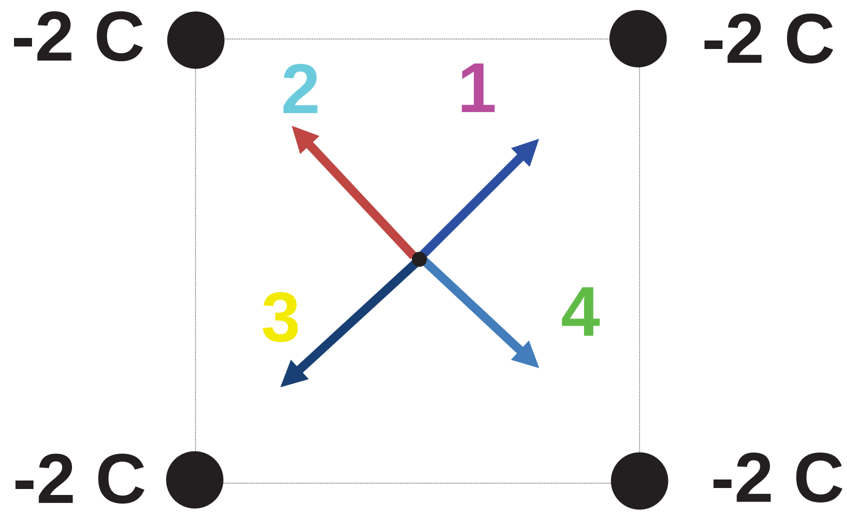
1. What is the electric field at the center of the square?

a. $4 \times K \times 2 / r^2$

b. $-4 \times K \times 2 / r^2$

c. Zero

d. $K \times 2 / r^2$

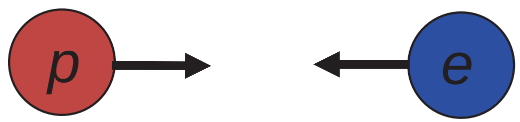


2. A proton and an electron are held apart a distance of 1 m and then let go. Where would they meet?

1) in the middle

2) closer to the electron’s side

3) closer to the proton’s side

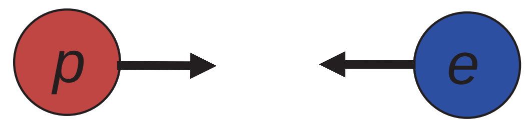


2. A proton and an electron are held apart a distance of 1 m and then let go. Where would they meet?

1) in the middle

2) closer to the electron's side

<u>3) closer to the proton's side</u>



By Newton's 3rd law, the electron and proton feel the **same force**. But, since $F = ma$, and since the **proton's mass is much greater**, the **proton's acceleration will be much smaller**!

Thus, they will meet **closer to the proton's original position**.

Continuous distribution of charges

- A continuous distribution of charge may be treated as a succession of point charges. The total field is then the integral of the point fields due to each bit of charge:

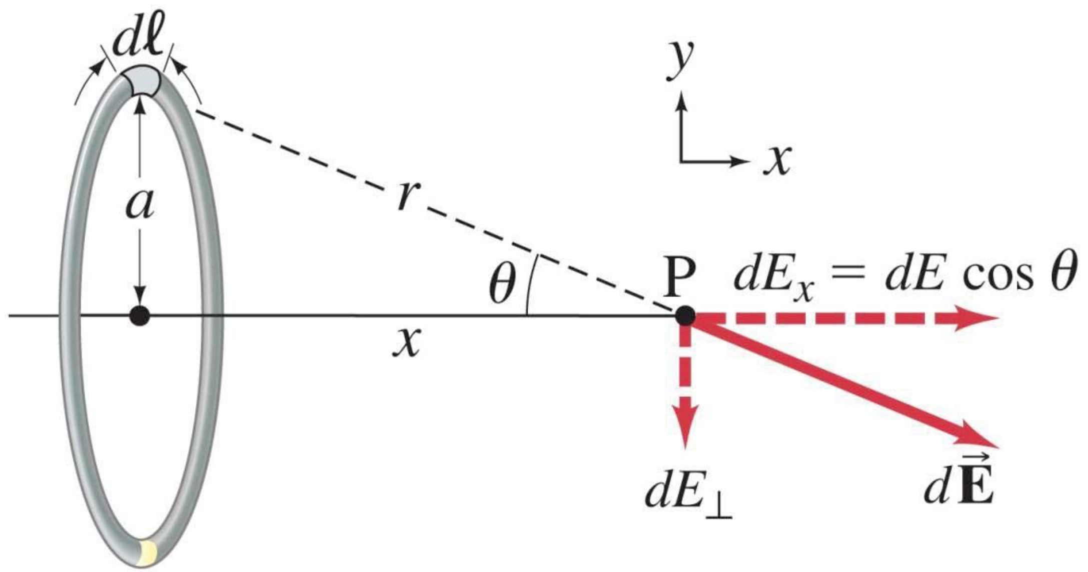
$$E = \int dE = \int k\,dq / r^2$$

- dq can be expressed in various forms

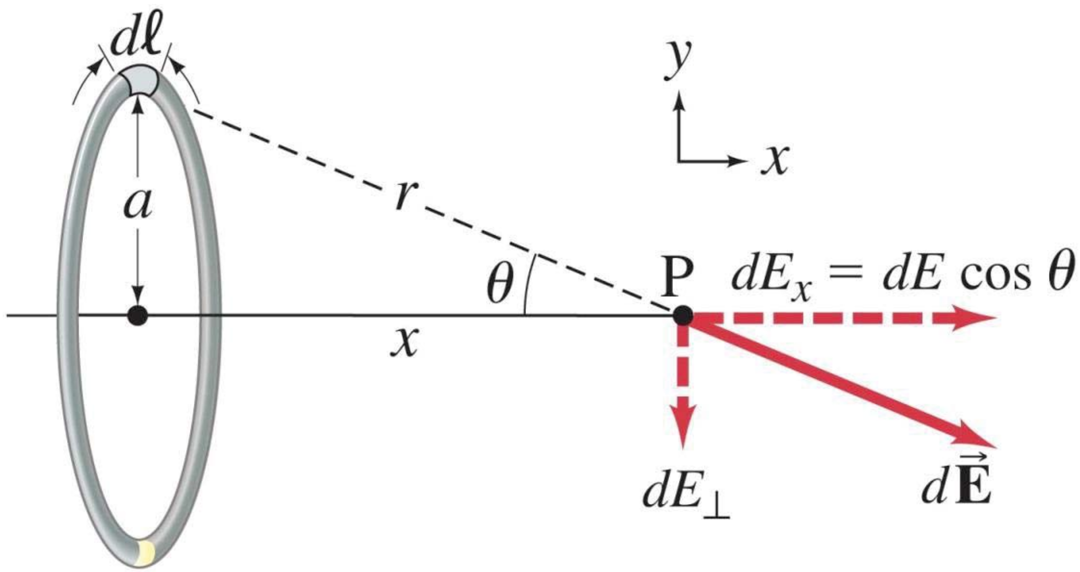
1. For ring: $dq = \frac{Q}{\text{Circumference}}\, dl = \lambda\, dl$ $\quad C/m$
2. For circular disk : $dq = \frac{q}{\text{Area}}\, dA = \sigma\, dA$ $\quad C/m^2$
3. For ball : $dq = \frac{q}{\text{Volume}}\, dV = \rho\, dV$ $\quad C/m^3$

Example 06

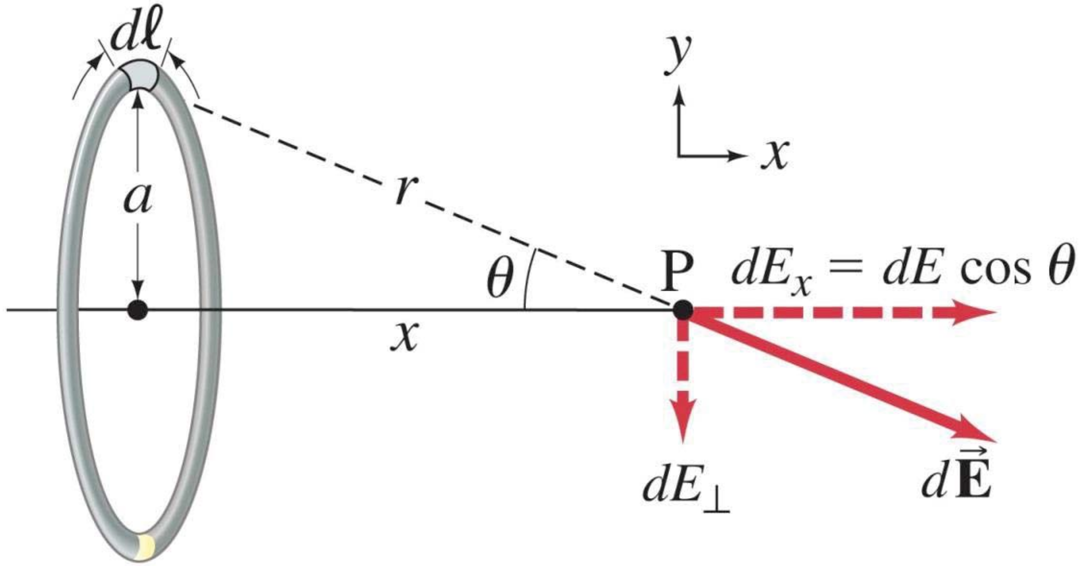
- A thin, ring-shaped object of radius a holds a total charge $+Q$ distributed uniformly around it. Determine the electric field at a point P on its axis, a distance x from the center. Let λ be the charge per unit length (C/m).



- The E_y components = 0 due to symmetry
- The E_x component will be calculated
- $\cos\theta = \frac{x}{(a^2 + x^2)^{1/2}}$
- $E_x = \int_0^{2\pi a} \frac{k}{r^2} \cos\theta \, dq$
- $E_x = \int_0^{2\pi a} \frac{k\,\lambda \cos\theta}{r^2} \, dl$
- $E_x = \int_0^{2\pi a} \frac{k\,\lambda\, x}{r^2 (a^2 + x^2)^{1/2}} \, dl$
- $r^2 = a^2 + x^2$

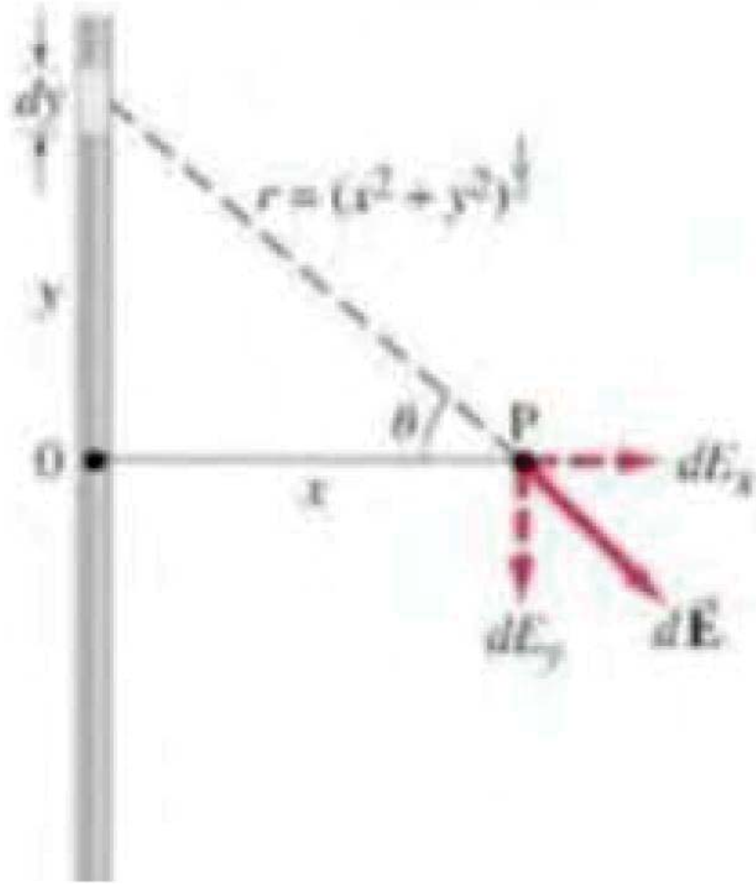


- $E_x = \int_0^{2\pi a} \frac{k\,\lambda\,x}{(a^2+x^2)^{3/2}}\, dl$
- $E_x = \frac{k\,\lambda\,x}{(a^2+x^2)^{3/2}} \int_0^{2\pi a} dl$
- $E_x = \frac{2\,\pi\,k\,a\,\lambda\,x}{(a^2+x^2)^{3/2}}$
- $\lambda = \frac{q}{2\,\pi\,a} \rightarrow q = 2\,\pi\,a\,\lambda$
- $E_x = \frac{k\,q\,x}{(a^2+x^2)^{3/2}}$

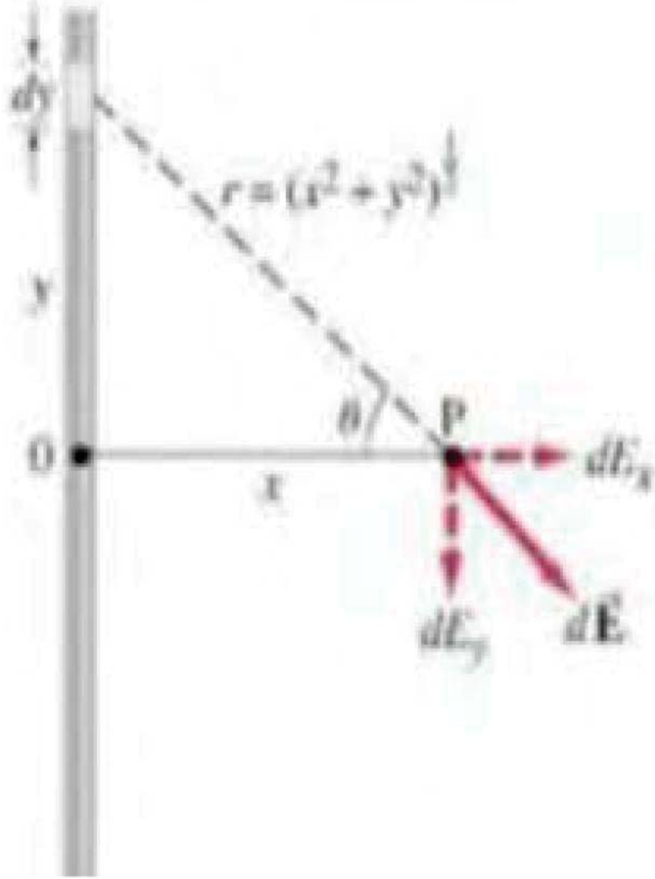


Example 07

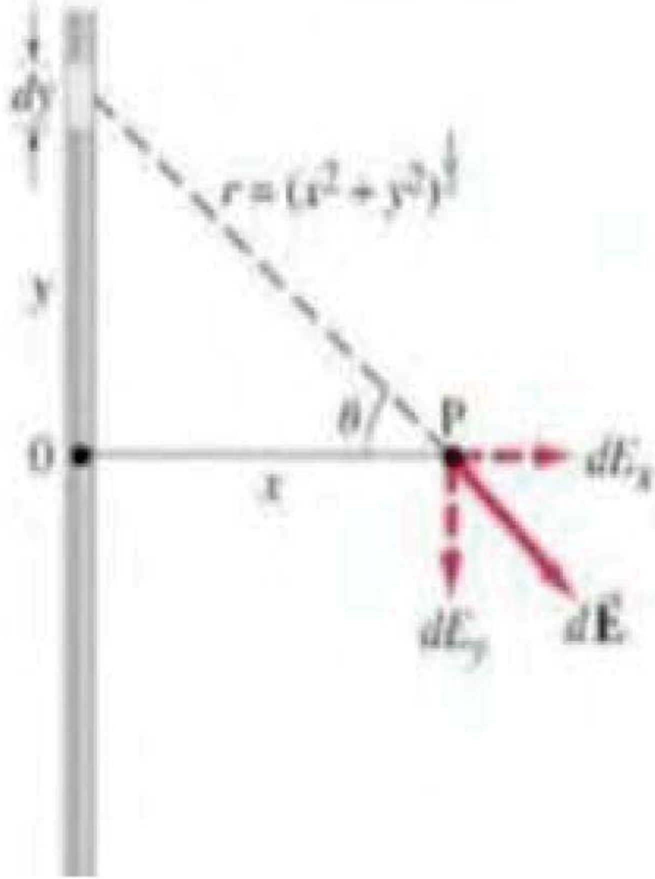
- Determine the magnitude of the electric field at any point P a distance *x* from the midpoint 0 of a very long line (a wire, say) of uniformly distributed positive charge. Assume *x is* much smaller than the length of the wire, and let A be the charge per unit length (C/m).



- $E_y = 0$ *due to summetry*
- $E_x = E \cos \theta$
- $E_x = k \, \frac{dq}{r^2} \cos \theta$
- $dq = \lambda \, dy$ & $\quad r^2 = x^2 + y^2$
- $dE_x = k \, \frac{\lambda \cos \theta \, dy}{x^2 + y^2}$
- $\cos \theta = \frac{x}{(x^2 + y^2)^{1/2}} \rightarrow \frac{1}{x^2 + y^2} = \frac{\cos \theta^2}{x^2}$
- $dE_x = \frac{k \, \lambda \, \cos \theta^3 \, dy}{x^2}$

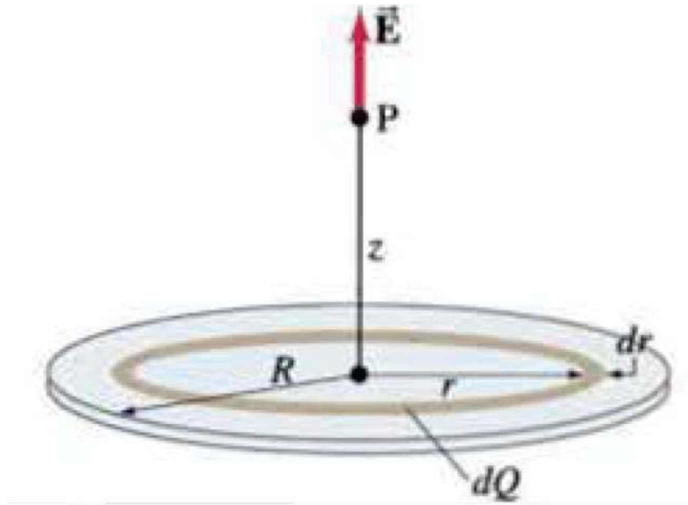


- $\tan\theta = \frac{y}{x} \rightarrow y = x\tan\theta$
- $dy = x\,d\theta\ \sec\theta^2 = \frac{x}{\cos\theta^2}\ d\theta$
- $as\ dE_x = \frac{k\,\lambda\ \cos\theta^3\ dy}{x^2}\frac{x}{\cos\theta^2}\ d\theta$
- $dE_x = \frac{k\,\lambda\cos\theta\ d\theta}{x}$
- $E_x = \frac{k\,\lambda}{x}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\cos\theta\ \ d\theta$
- $E_x = \frac{k\,\lambda}{x}(\sin\frac{\pi}{2} - \sin\frac{-\pi}{2}) = 2\frac{k\,\lambda}{x}$

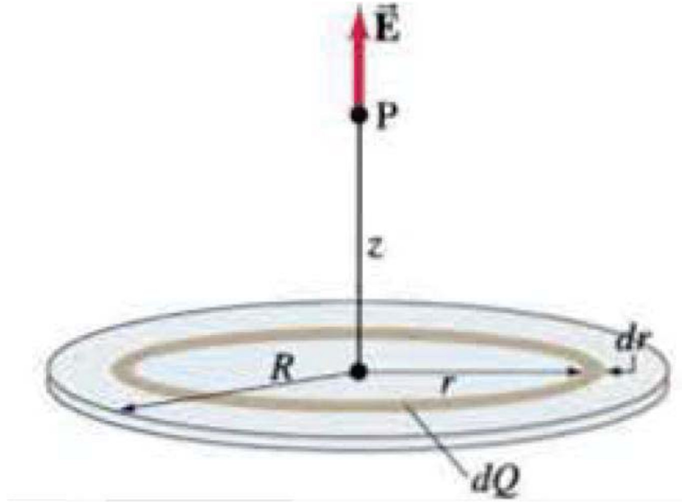


Example 08 (21-12)

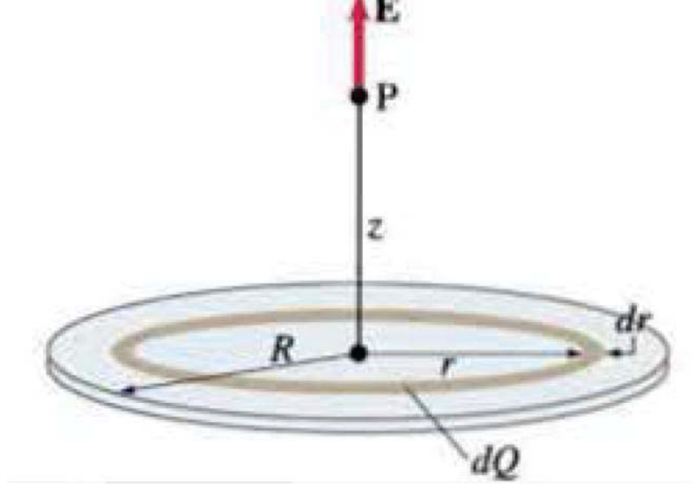
- Uniformly charged disk. Charge is distributed uniformlyover a thin circular disk of radius *R.* The charge per unit area (C/m2) is *u.* Calculate the e lectric field at a point P on the axis of the disk, a distance 4 above its center



- Electricity field for ring (example 06) :
- $dE = k \frac{dQ\, Z}{(r^2+z^2)^{3/2}}$
- $dQ = \sigma\, dA$
- $Area = 2\, \pi\, r\, dr$
- $dQ = \sigma\, 2\, \pi\, r\, dr$
- $dE = 2\, \pi\, \sigma\, Z\, k \frac{r\, dr}{(r^2+z^2)^{3/2}}$
- $E = 2\, \pi\, \sigma\, Z\, k \int_0^R \frac{r\, dr}{(r^2+z^2)^{3/2}}$
- $E = 2\, \pi\, \sigma\, Z\, k \left[\frac{-1}{(r^2+z^2)^{1/2}}\right]_0^R$



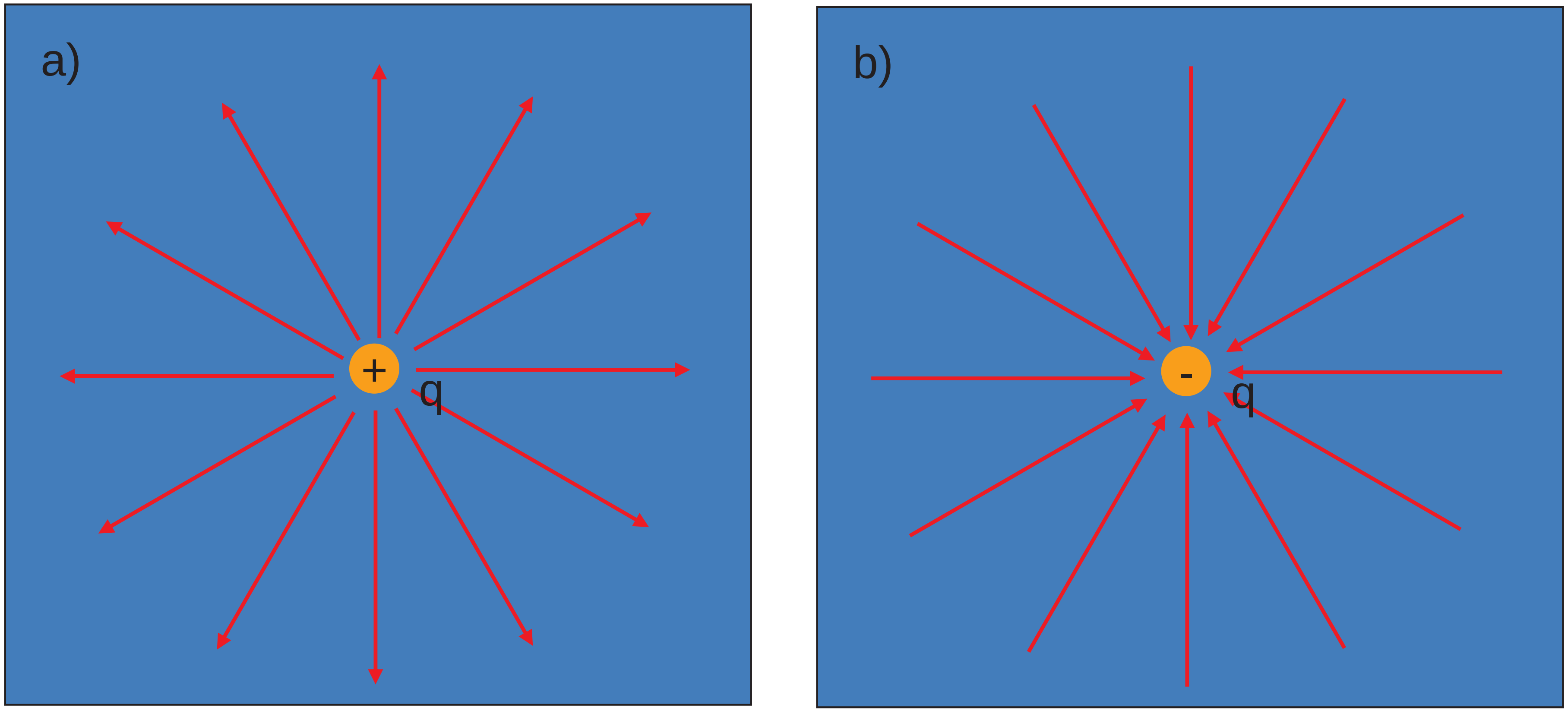
- $E = 2\,\pi\,Z\,\sigma\,k\,[\frac{-1}{(R^2+z^2)^{\frac{1}{2}}} + \frac{1}{z}]$
- $E = 2\,\pi\,\sigma\,k\,[1 - \frac{1}{(R^2+z^2)^{\frac{1}{2}}}]$



Electric Field Lines

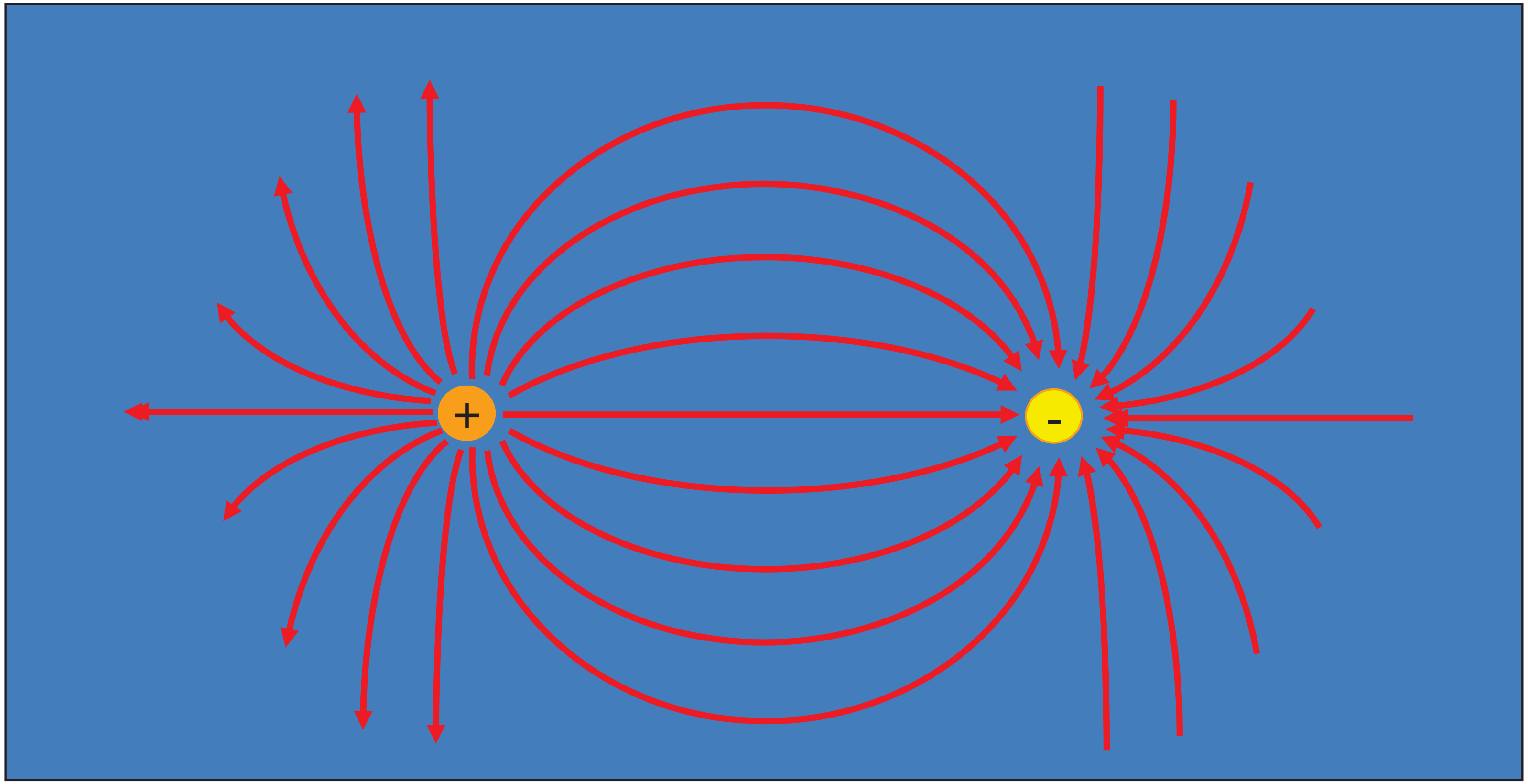
- A convenient way to visualize field patterns is to draw lines in the direction of the electric field.
- Such lines are called ***field lines***.
- Remarks:
  1. Electric field vector, E, is tangent to the electric field lines at each point in space.
  2. The number of lines per unit area through a surface perpendicular to the lines is proportional to the strength of the electric field in a given region.
- E is large when the field lines are close together and small when far apart.

- Electric field lines of single positive (a) and (b) negative charges.

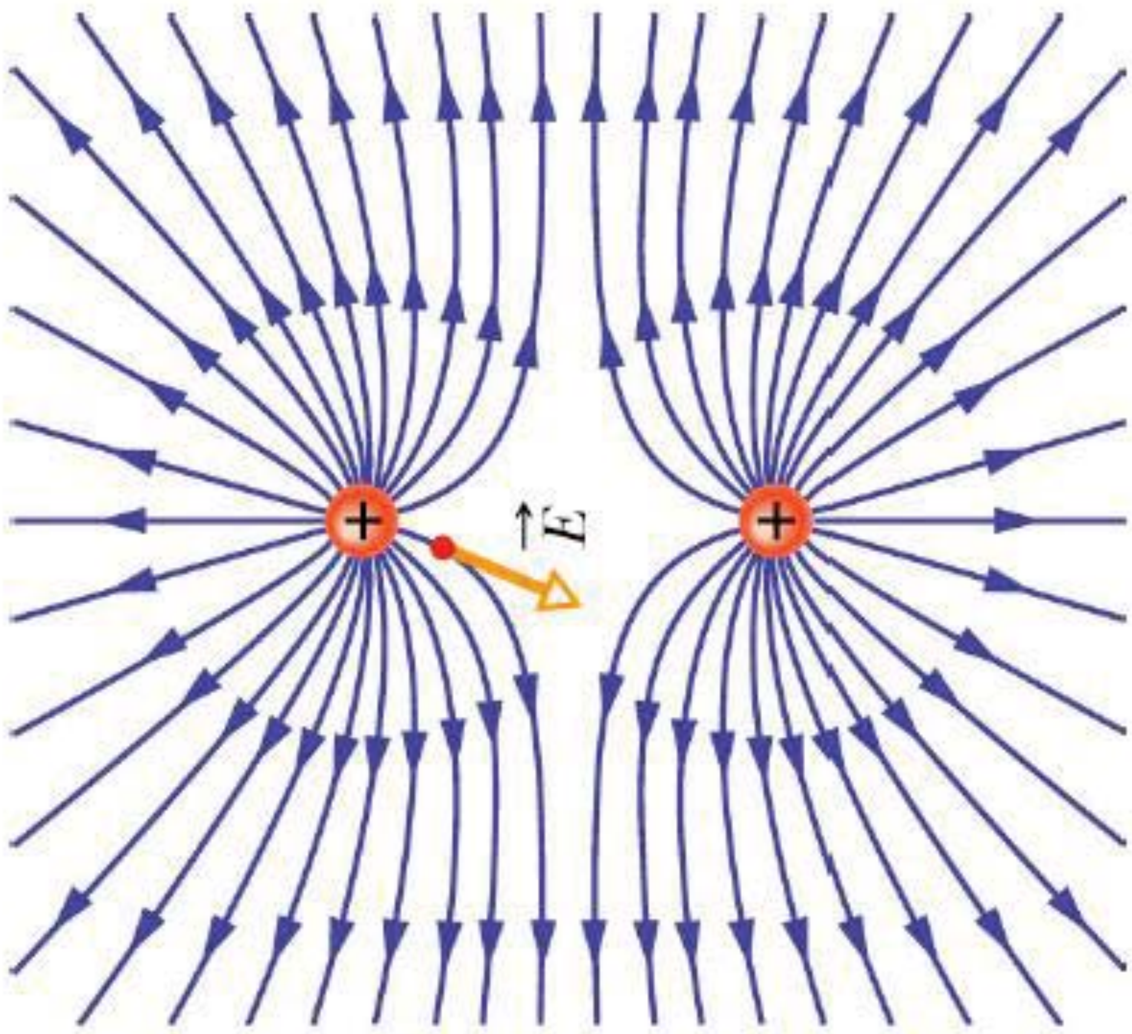


- Rules for drawing electric field lines for any charge distribution.
    1. Lines must <u>begin</u> on <u>positive</u> charges (or at <u>infinity</u>) and must <u>terminate</u> on <u>negative</u> charges or in the case of excess charge at <u>infinity</u>.
    2. The <u>number</u> of <u>lines</u> drawn leaving a positive charge or approaching a negative charge is <u>proportional</u> to the <u>magnitude</u> of the <u>charge</u>.
    3. <u>No</u> <u>two</u> field <u>lines</u> can <u>cross</u> each other.

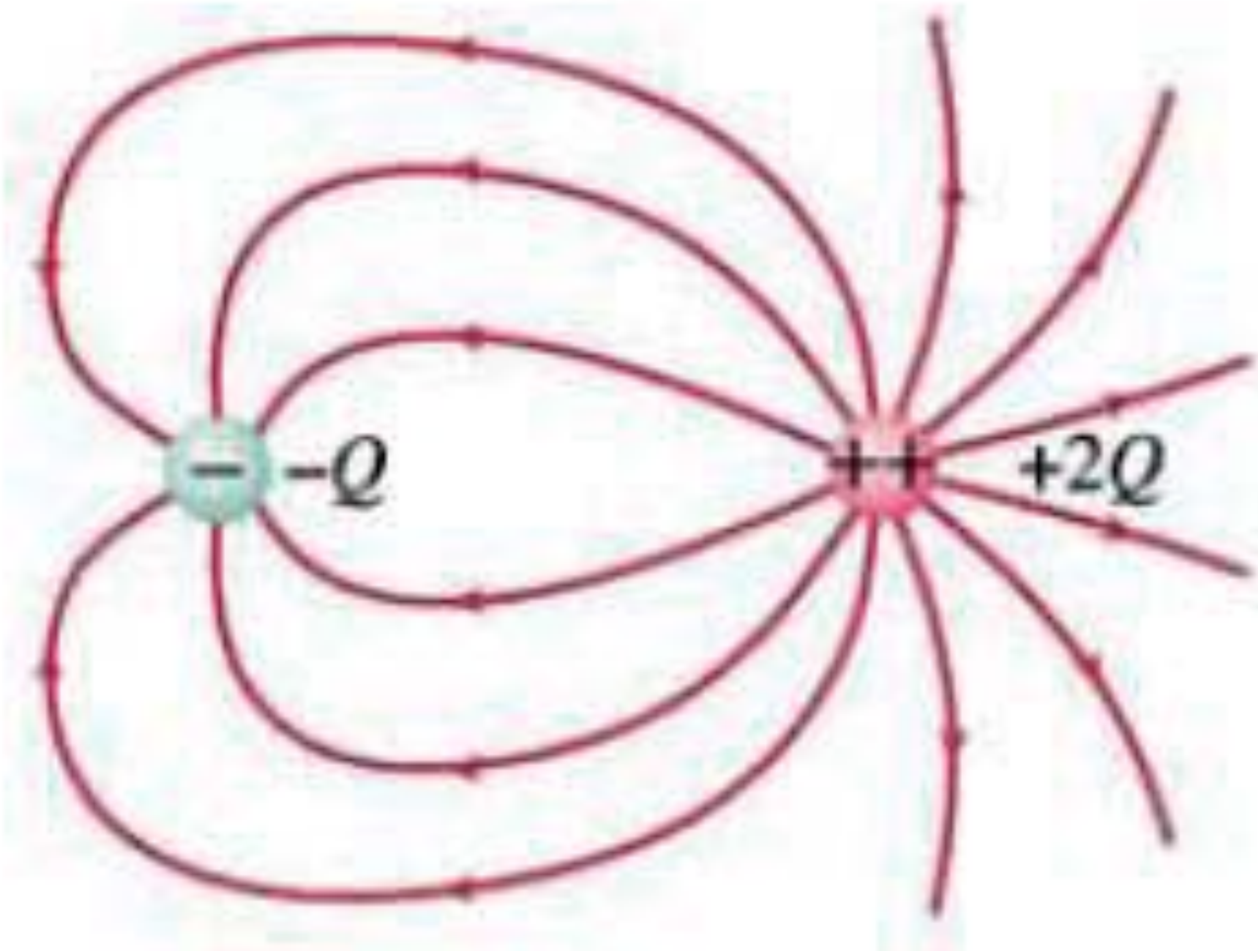
- Electric field lines of a ***dipole***.



•

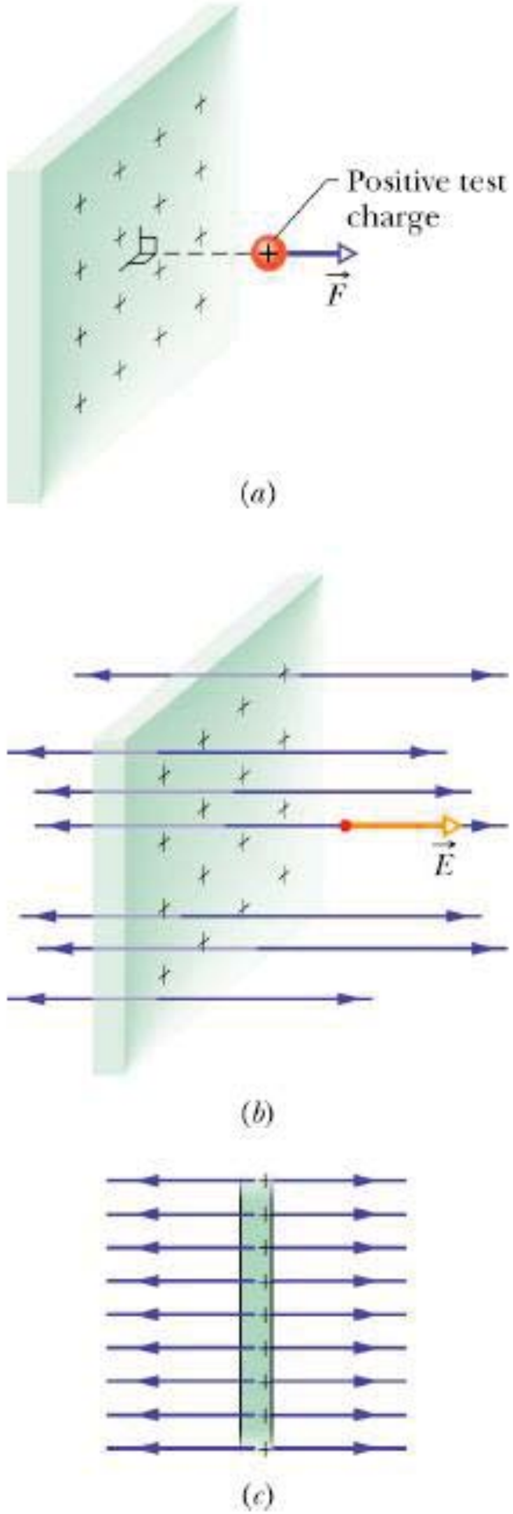


E



−
−Q
++
+2Q

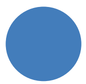
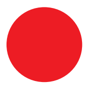
- Example of field lines for a uniform distribution of positive
- charge on one side of a very large nonconducting sheet.
- This is called a uniform electric field.

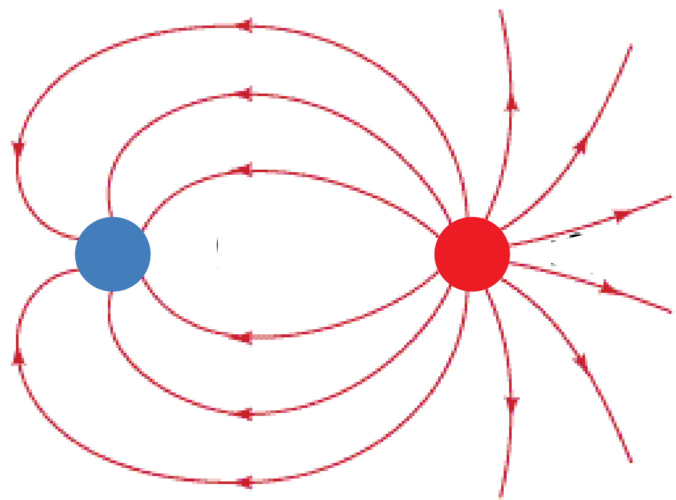


(a) (b) (c)

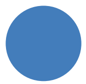
MCQ

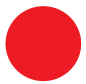
1. Which of the charges has the greater magnitude?

1) 
2) 
3) both the same

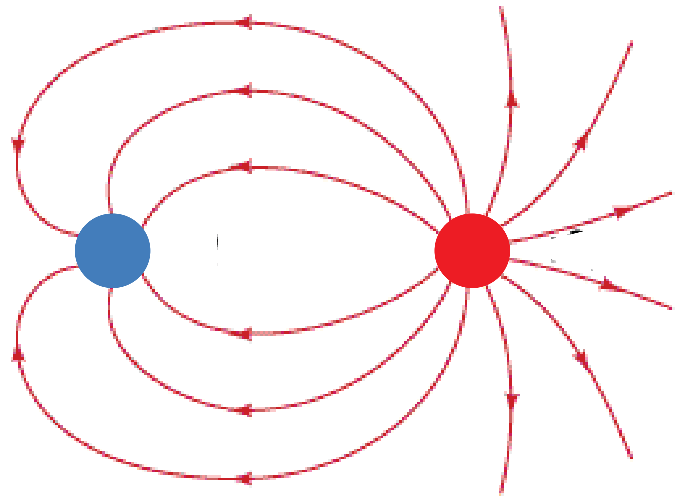


1. Which of the charges has the greater magnitude?

1) 

<u>2)</u> 

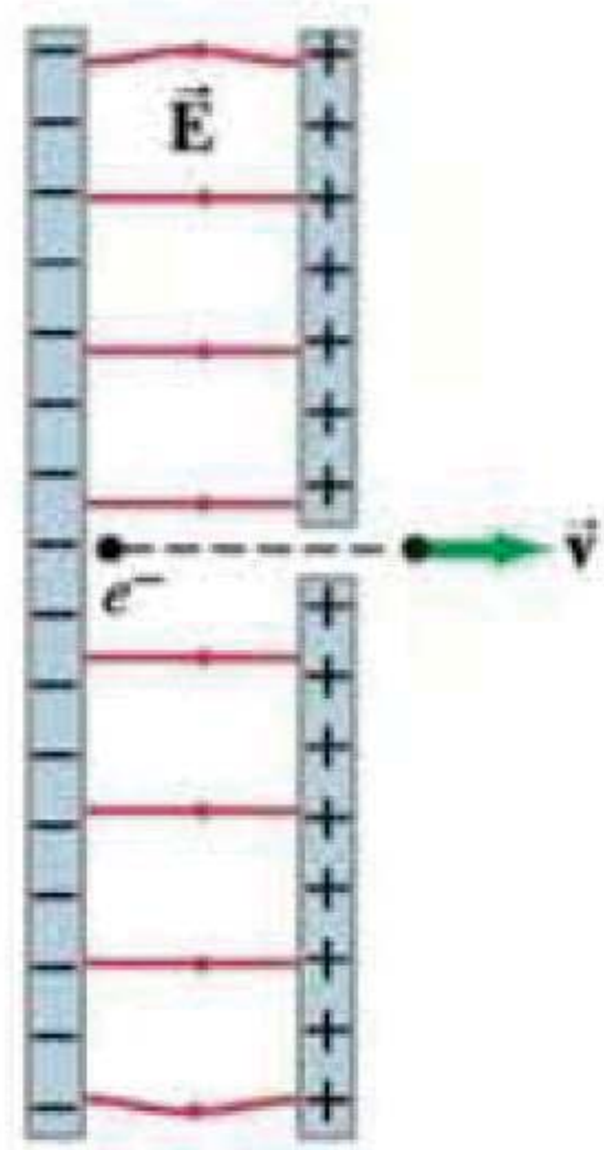
3) both the same



The field lines are **denser around the red charge**, so the **red one has the greater magnitude**.

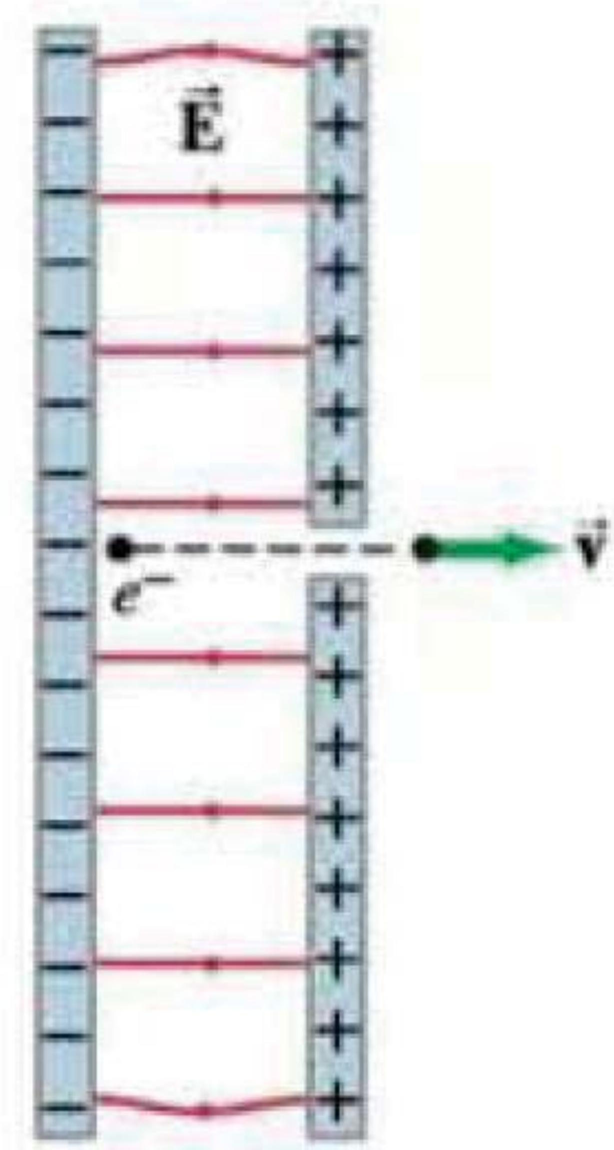
Motion of a Charged Particle in an Electric Field

- $Force = Field \times Charge\ Quantiry$
  - $F = E \times Q$
- $Acceleration = \frac{Force}{Mass}$
  - $a = \frac{F}{m} = \frac{E \times Q}{m}$
- $Velocity = Acceleration \times time$
  - $v = a \times t = \frac{E \times Q \times t}{m}$
- $distance = velocity \times time$
  - $d = v \times t = \frac{E \times Q \times t^2}{m}$



Example 10

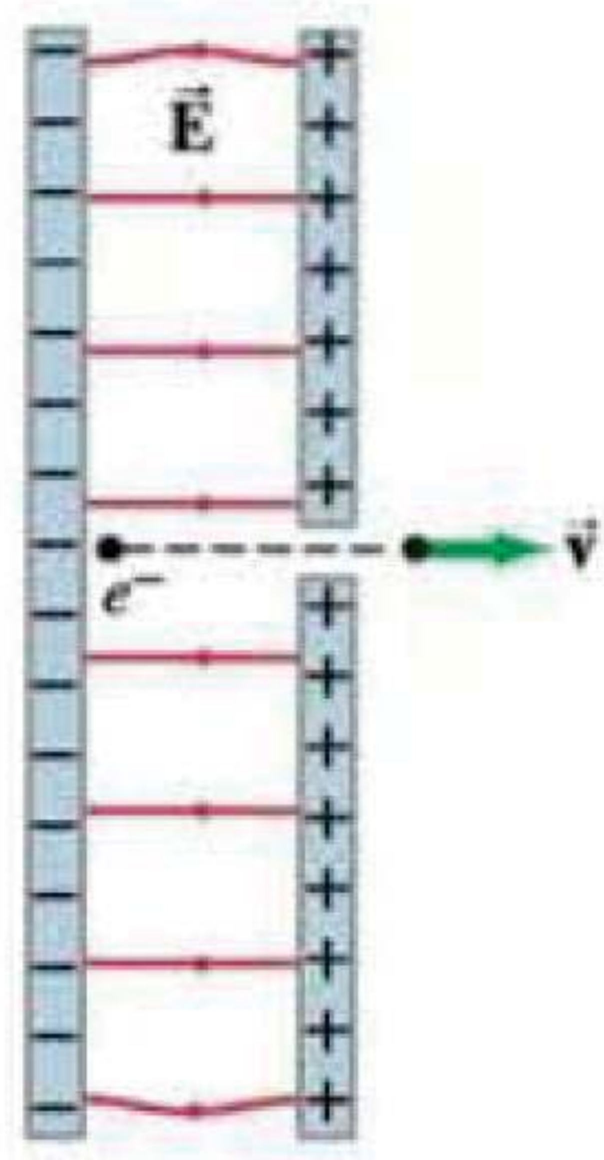
- An electron (mass $m = 9.1 \times 10^{-31}$ kg) is accelerated in the uniform field E (= 2.0×10^{4} N/ C) between two parallel charged plates. The separation of the plates is 1.5 cm. the electron is accelerated from rest near the negative plate and passes through a tiny hole in the positive plate
- (a) With what acceleration does it leave the hole?



m = 9.1 X 10^{-31} kg
E = 2.0 X 10^{4} N/ C
Q = 1.6 x 10^{-19} C

(a) $a = \frac{F}{m} = \frac{E \times q}{m} =$

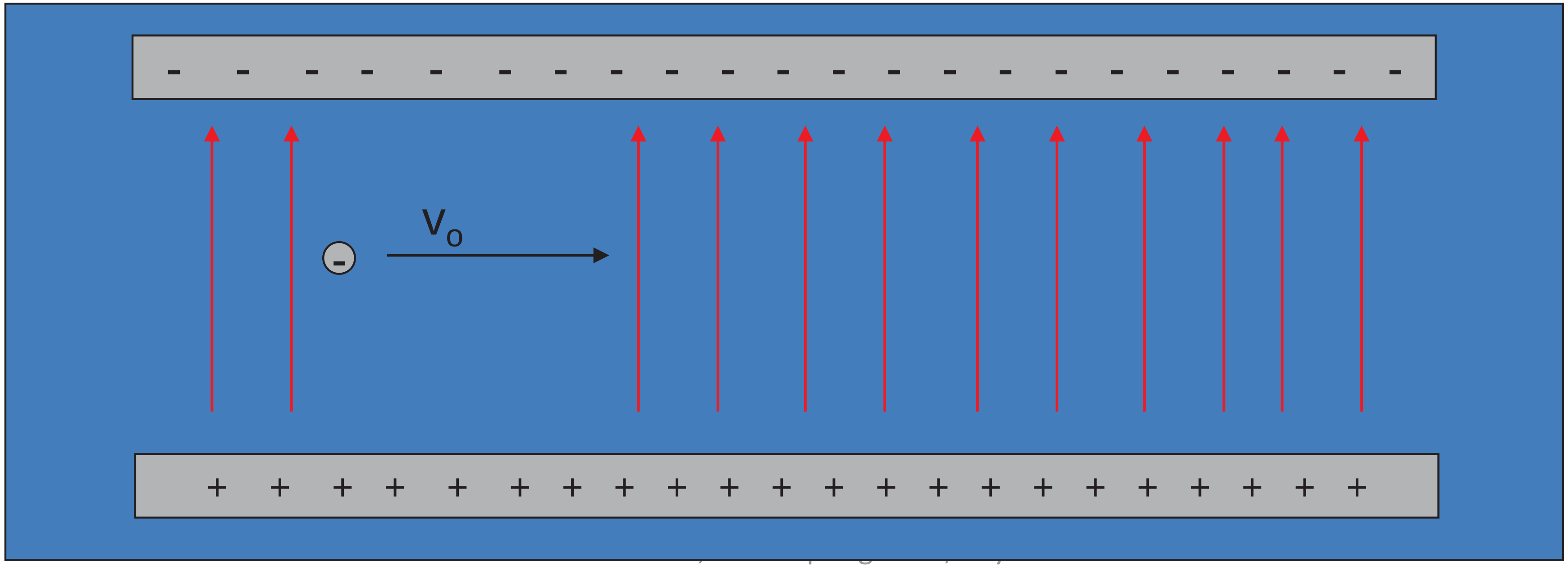
$= \frac{2 \times 10^{4} \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}} = 3.5 \text{ x } 10^{15}$



Example 11

Example:

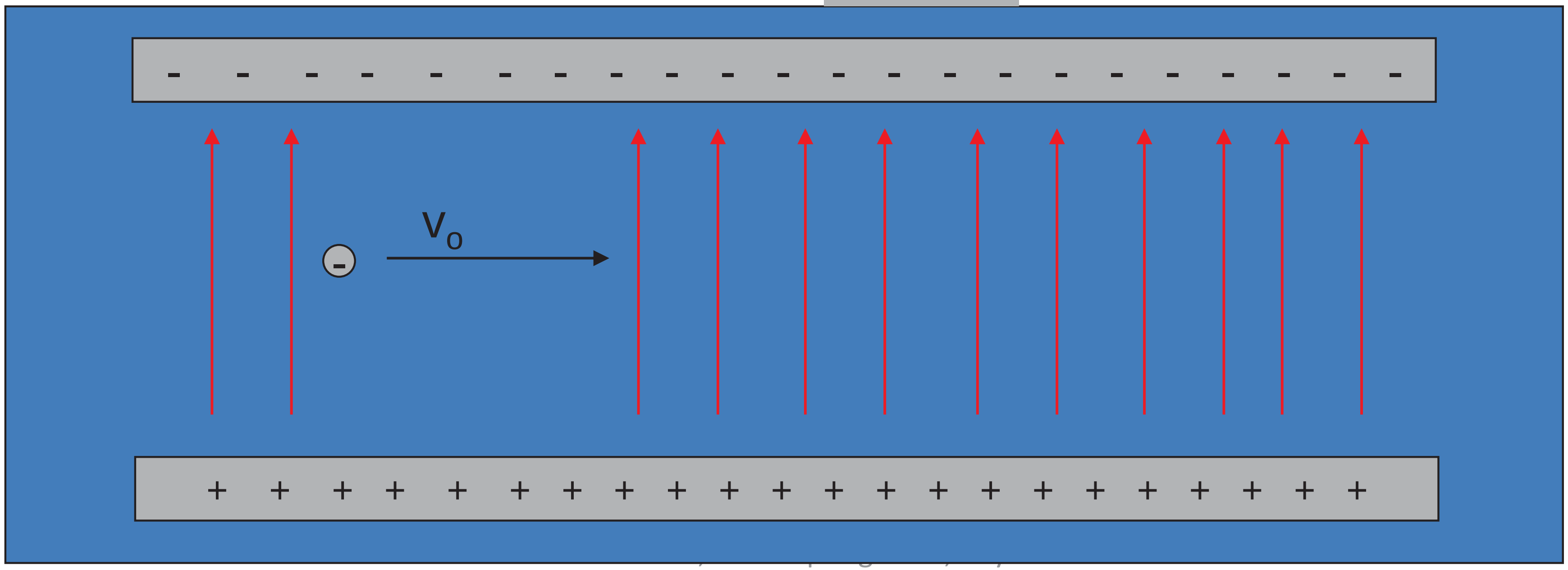
- An electron moving horizontally passes between two horizontal planes, the upper plane charged negatively, and the lower positively. A uniform, upward-directed electric field exists in this region. This field exerts a force on the electron. Describe the motion of the electron in this region.



Observations:

- Horizontally:
  - No electric field
  - No force
  - No acceleration
  - Constant horizontal velocity

$$E_x = 0$$
$$F_x = 0$$
$$a_x = 0$$
$$v_x = v_o$$
$$x = v_o t$$

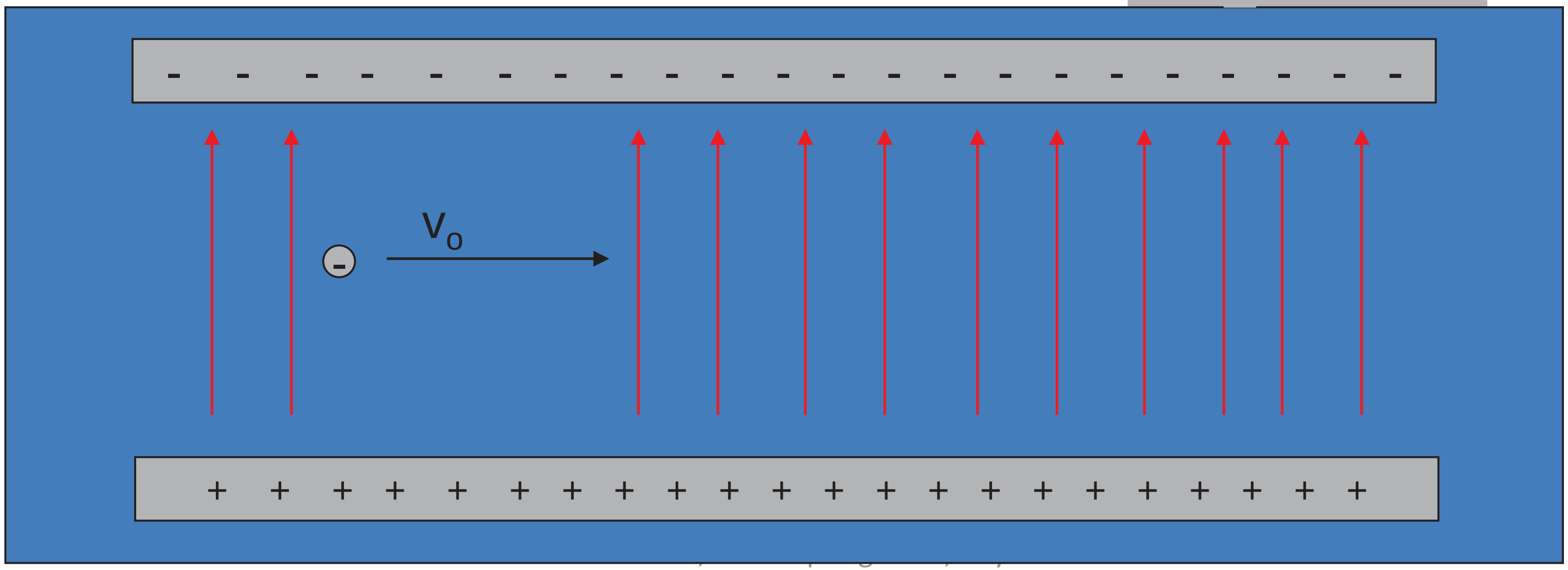


Newton's Second Law
$a = F_{net} / m$

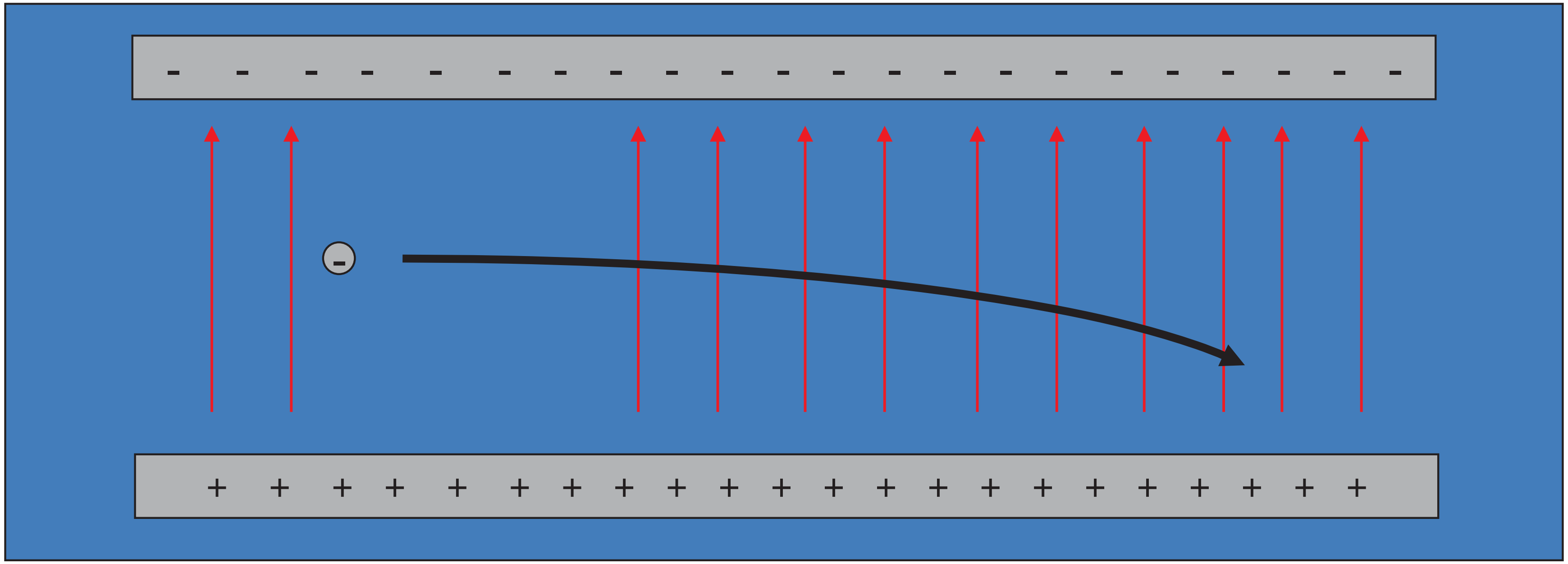
Observations:

- Vertically:
  - Constant electric field
  - Constant force
  - Constant acceleration
  - Vertical velocity increase linearly with time.

$$E_y = E_o$$
$$F_y = q_o E_o$$
$$a_y = q_o E_o / m_o$$
$$v_y = q_o E_o t / m_o$$
$$y = \cdot \quad q_o E_o t^2 / m_o$$



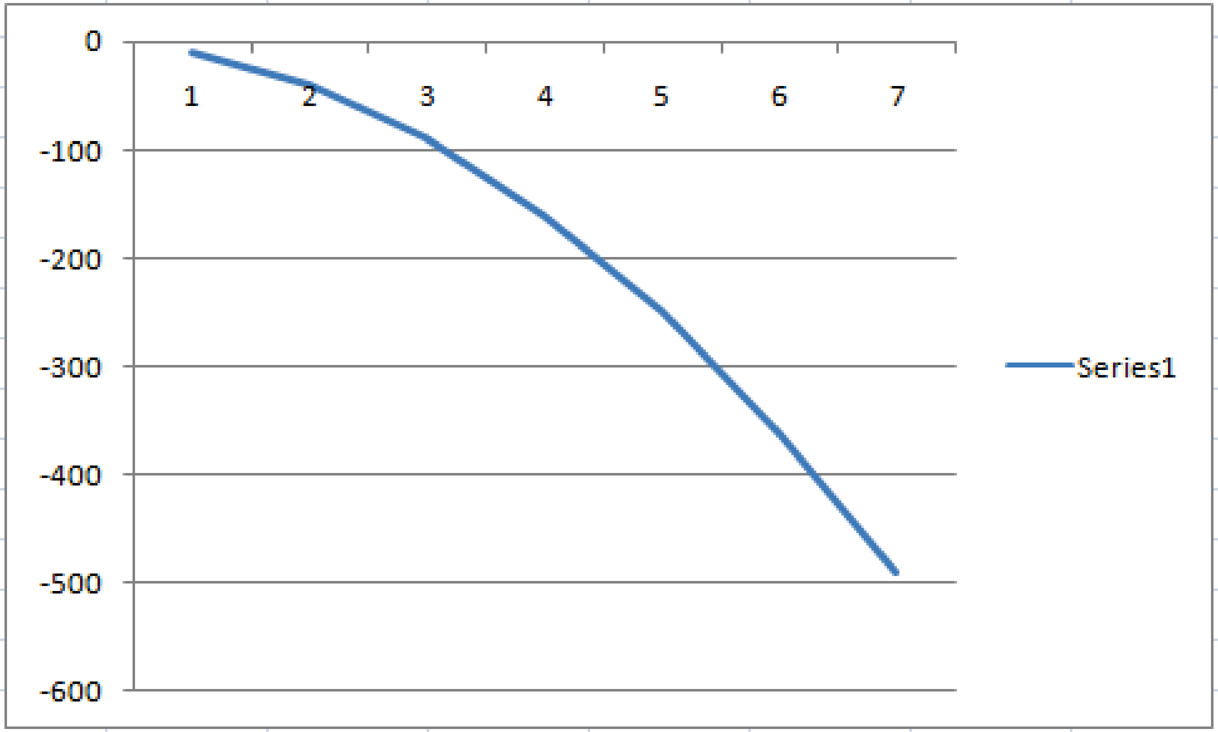
$y = constant\ x\ t^2$



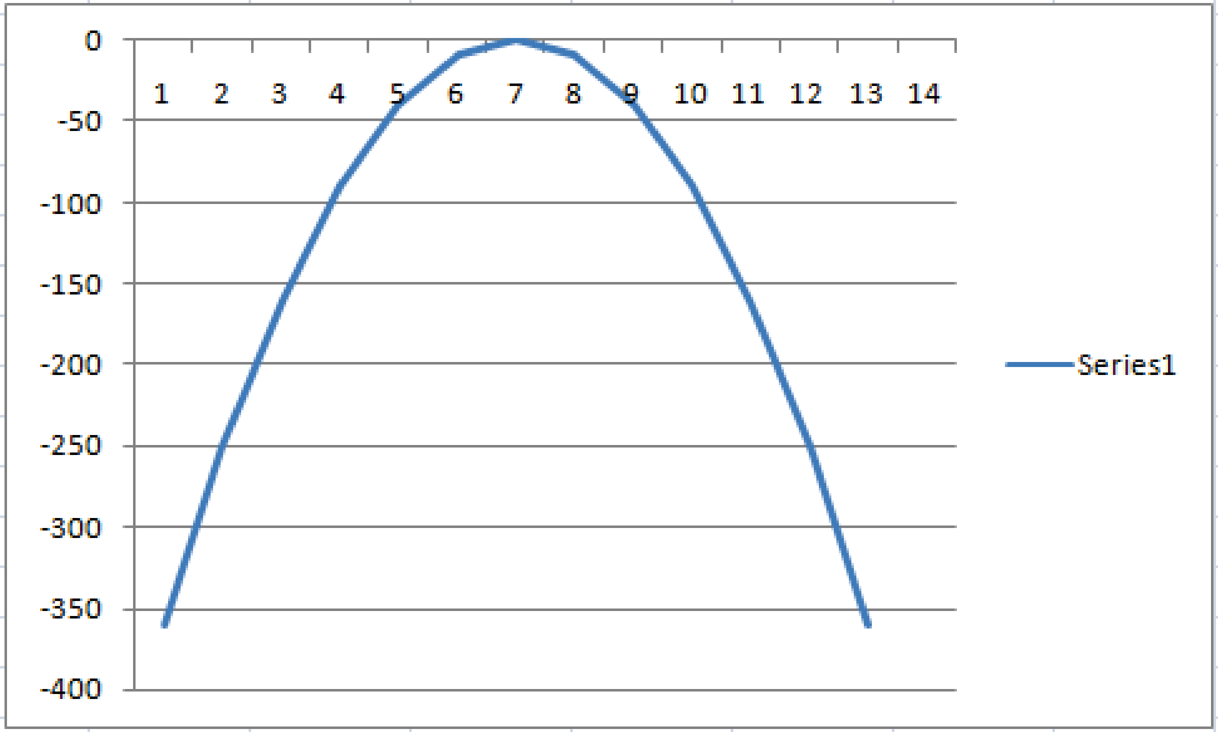
Conclusions:

- The charge will follow a parabolic path downward.
- Motion similar to motion under gravitational field only except the downward acceleration is now larger.

1	-10
2	-40
3	-90
4	-160
5	-250
6	-360
7	-490

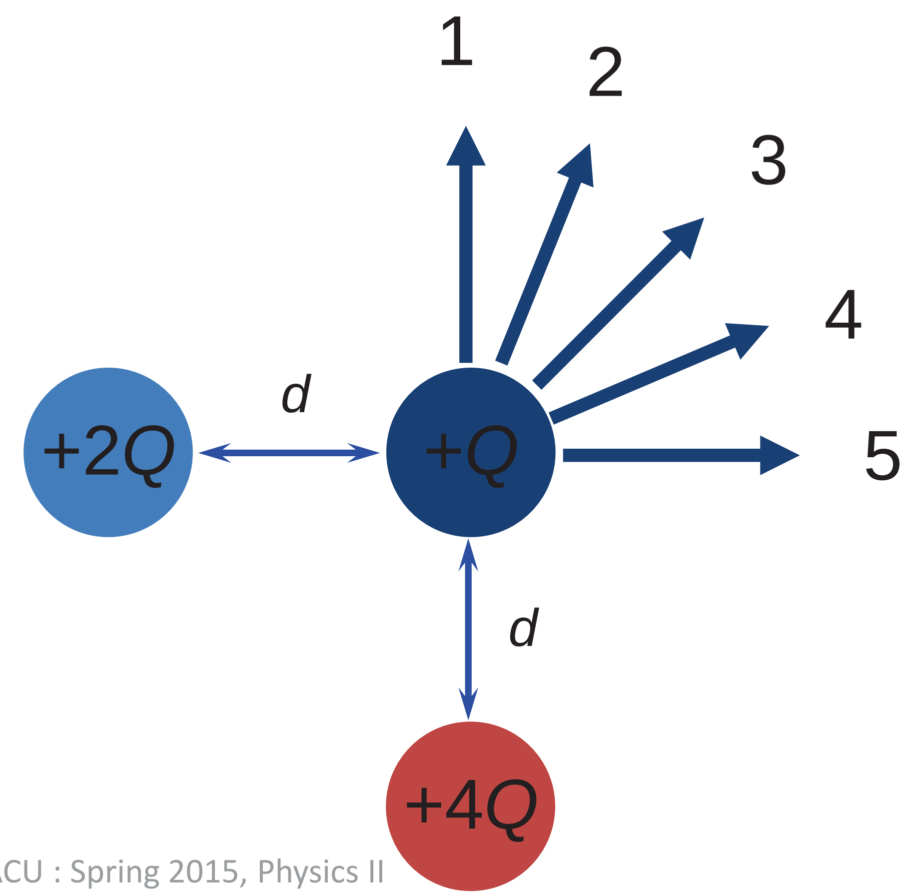


-6	-360
-5	-250
-4	-160
-3	-90
-2	-40
-1	-10
0	0
1	-10
2	-40
3	-90
4	-160
5	-250
6	-360

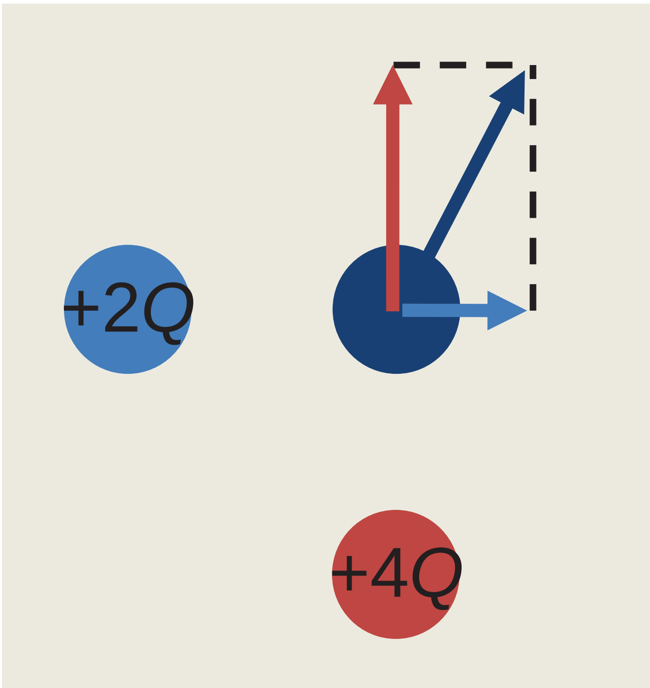


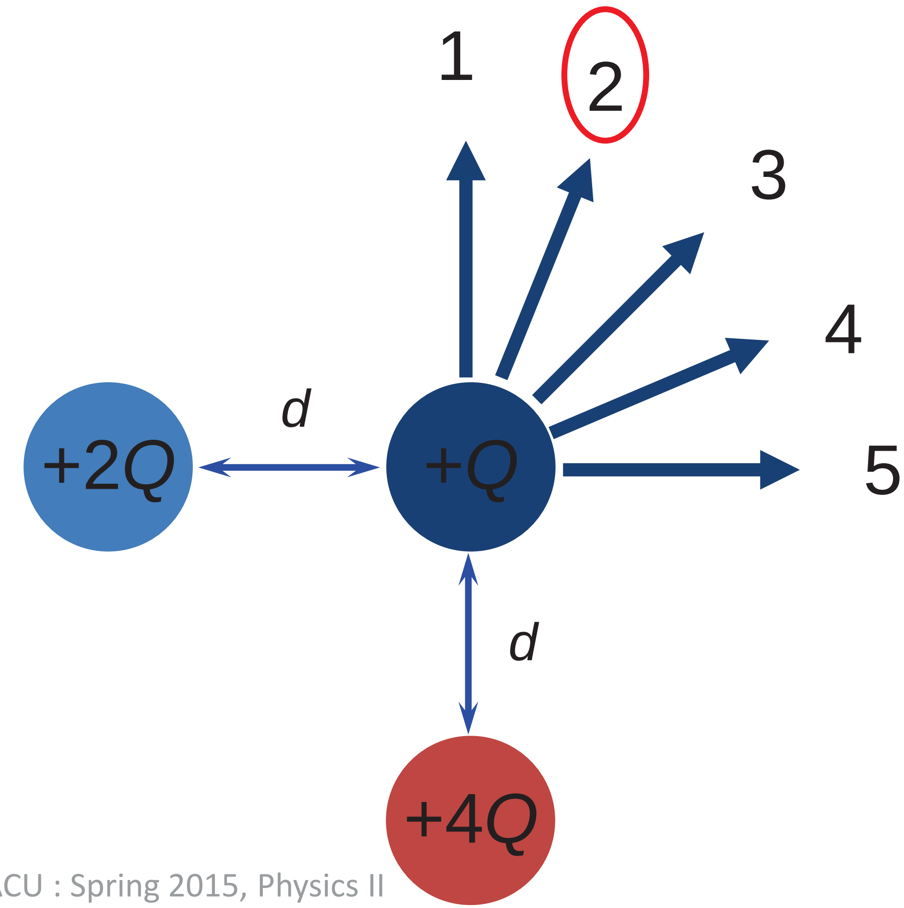
MCQ

- **Which of the arrows best represents the direction of the net force on charge +*Q* due to the other two charges?**



- **Which of the arrows best represents the direction of the net force on charge +*Q* due to the other two charges?**

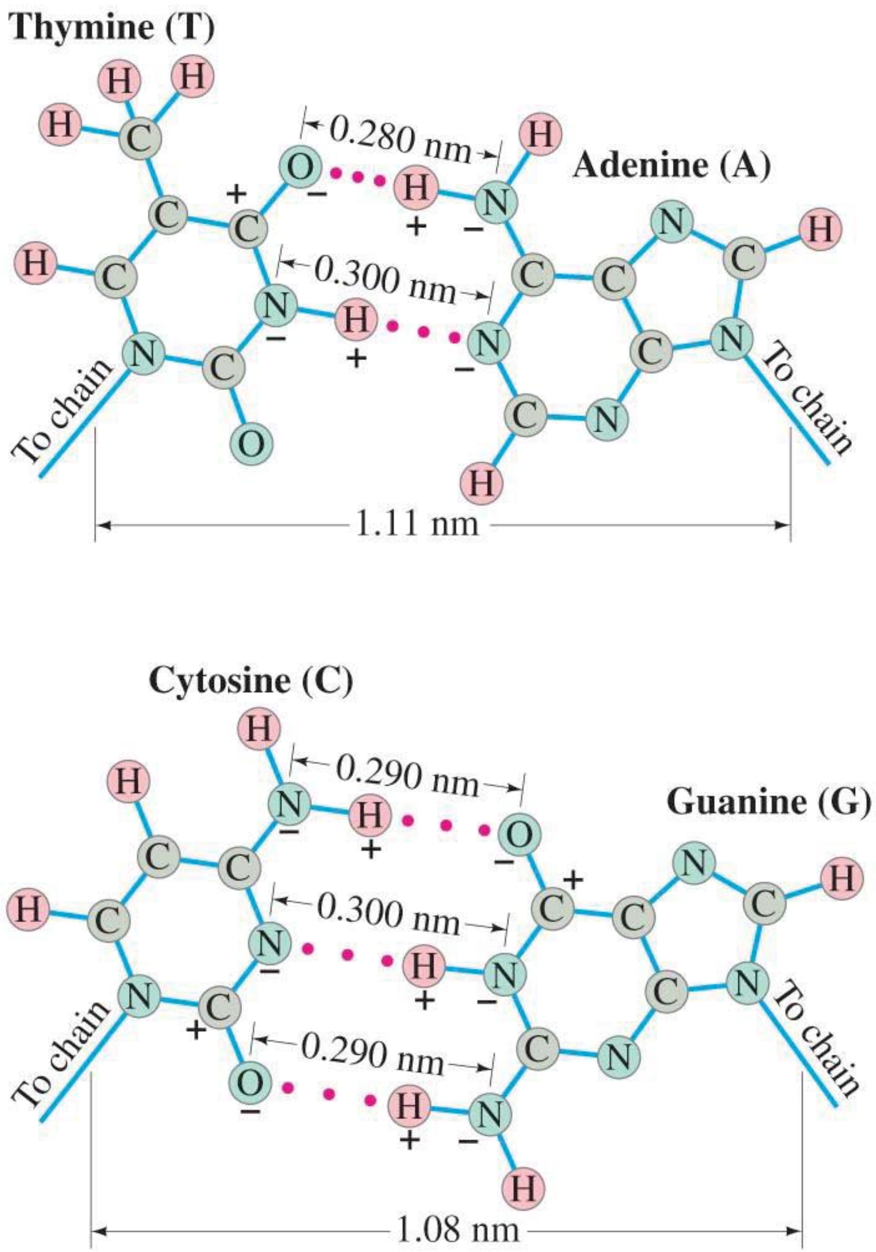




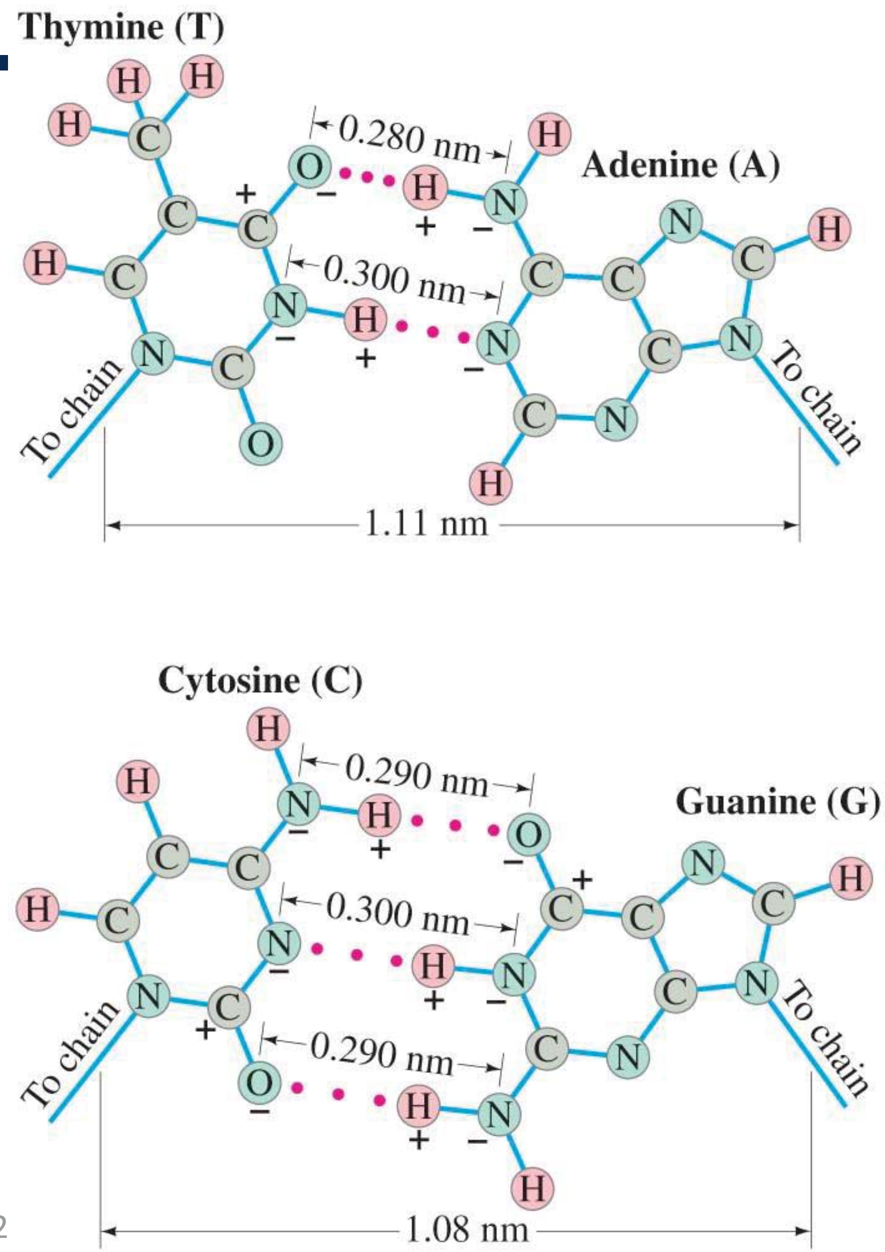
Electric Forces in Molecular Biology; DNA

- Molecular biology is the study of the structure and functioning of the living cell at the molecular level.
- The DNA molecule is a double helix:

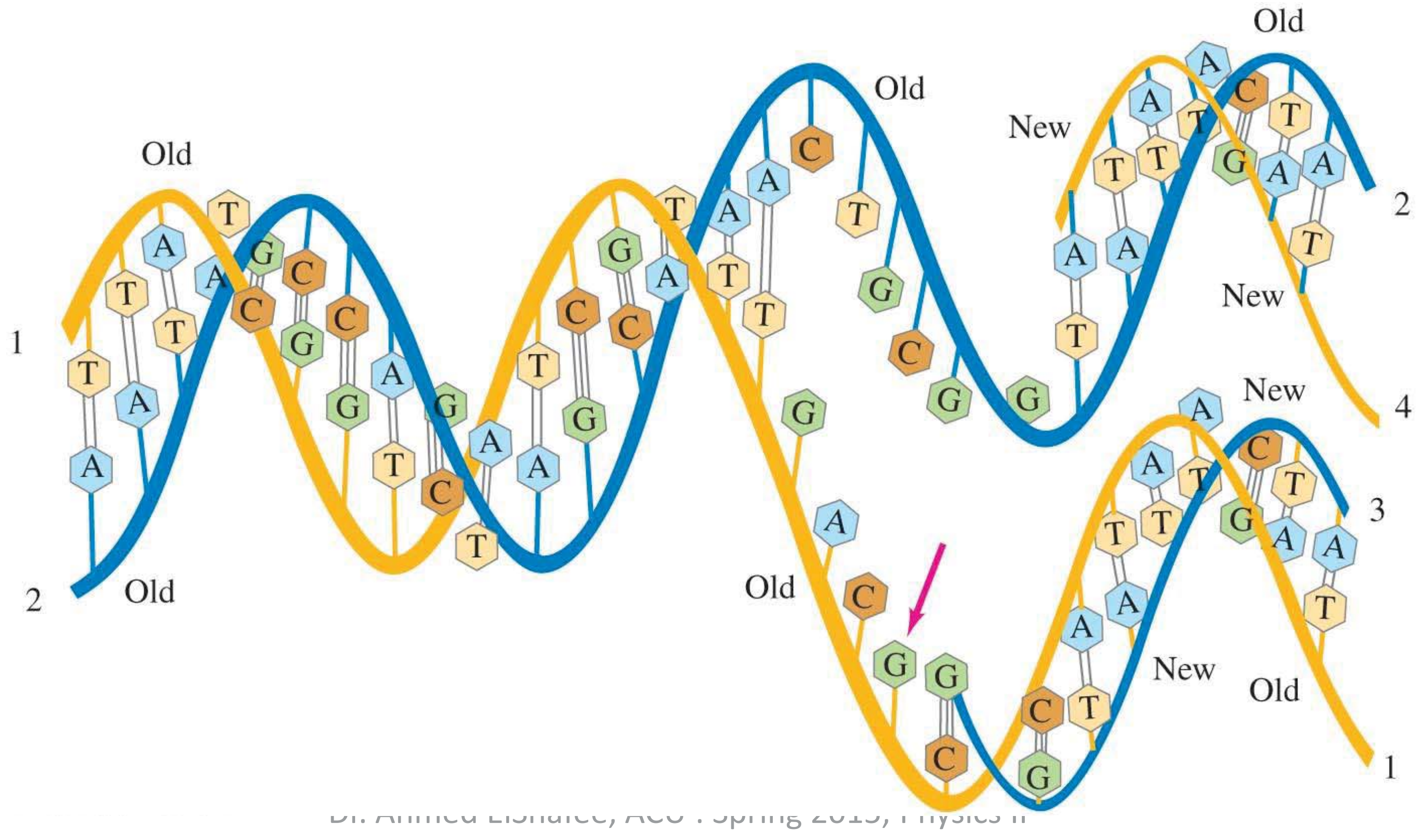
- The A-T and G-C nucleotide bases attract each other through electrostatic forces.
- Close-up” view of the helix, showing how A and T attract each other and how G and C attract each other through electrostatic forces.
- The + and - signs represent net charges, usually a fraction of e, due to uneven sharing of electrons.



- The red dots indicate the electrostatic attraction (often called a “weak bond” or “hydrogen bond”).
- Note that there are two weak bonds between A and T, and three between C and G.



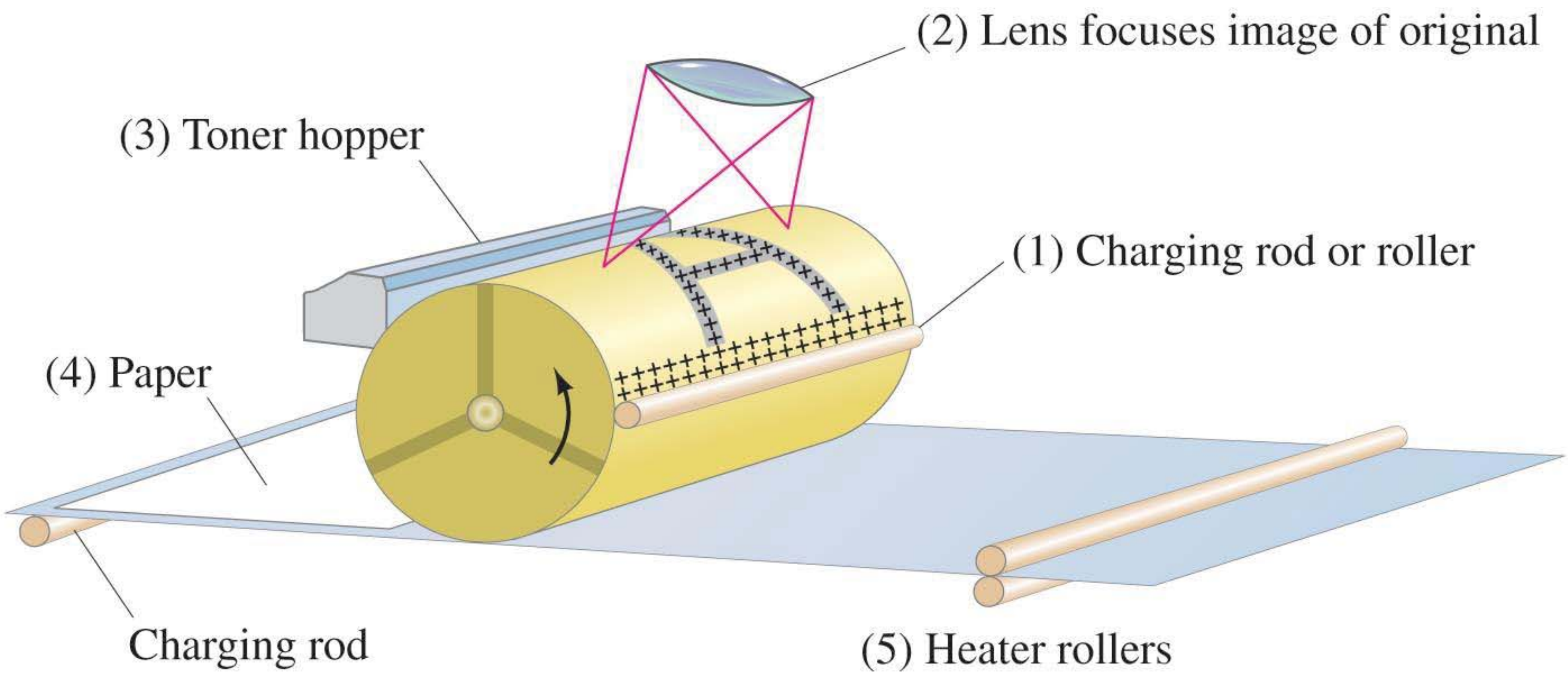
- Replication: DNA is in a “soup” of A, C, G, and T in the cell. During random collisions, A and T will be attracted to each other, as will G and C.



Photocopy Machines and Computer Printers Use Electrostatics

- project an image onto a special cylindrical drum.
- made of aluminum coated with a thin layer of selenium "photoconductivity", nonconductor in the dark, but a conductor when exposed to light.

•



(2) Lens focuses image of original
(3) Toner hopper
(1) Charging rod or roller
(4) Paper
Charging rod
(5) Heater rollers

- Step 1: placing of a uniform positive charge on the drum's selenium layer by a charged rod or roller, done in the dark.
- step 2, the image to be copied or printed is projected onto the drum. Written characters are dark, empty spaces are light

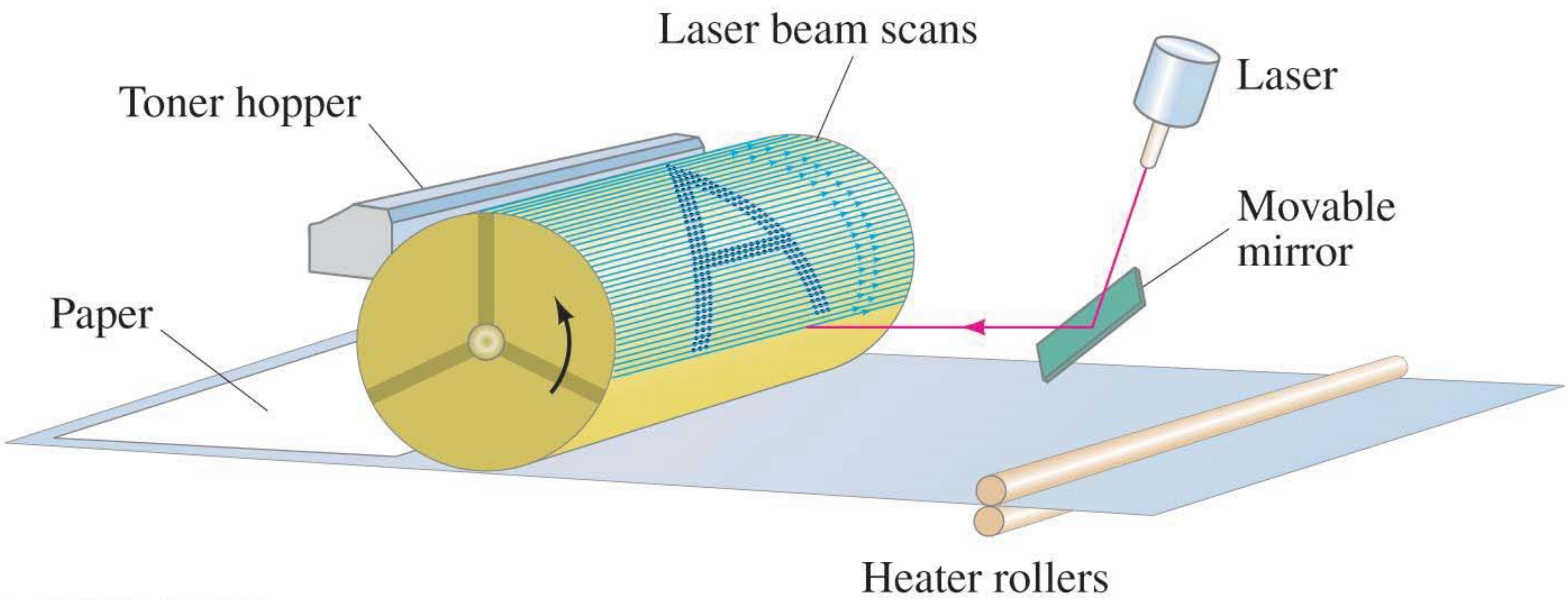
  Light area become conductors, and electrons flow from selenium layer to aluminum drum.
- Step 3, a fine dark powder known as *toner* is given a negative charge, and brushed on the drum as it rotates.

  The negatively charged toner particles arc attracted to the positive areas on the drum

- step 4, the rotating drum presses against a piece of paper which has been positively charged more strongly than the selenium, so the toner particles are transferred to the paper, forming the final image.
- Finally, step *5,* the paper is heated to fix the toner particles firmly on the paper.

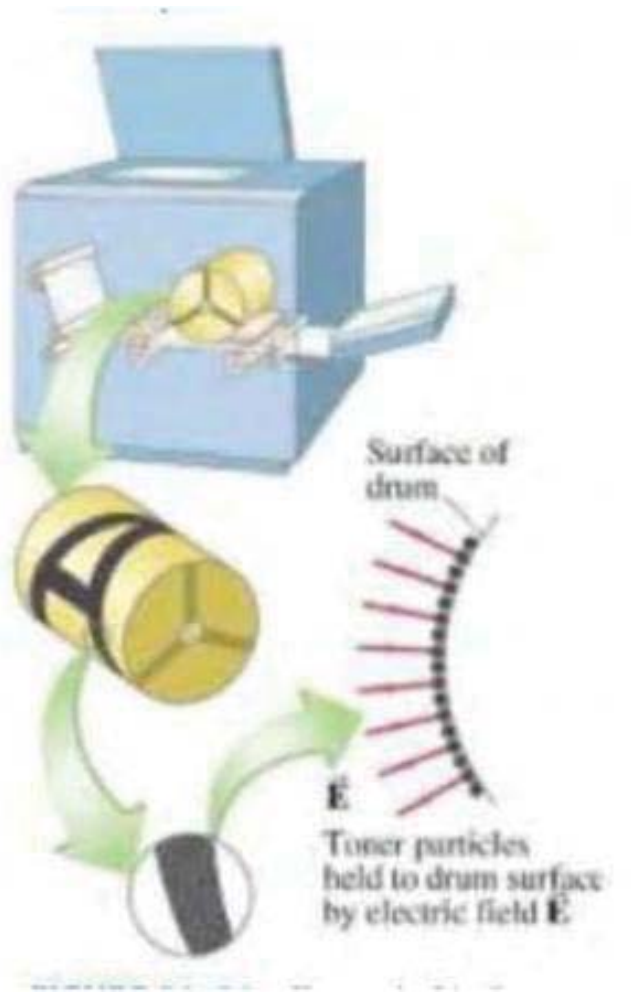
- In a color copier (or printer}, this process is repeated for each color- black, cyan (blue), magenta (red}, and yellow. Combining these Jour colors in different proportions produces any desired color

- A *laser primer,* on the other hand, uses a computer output to program the intensity of a laser beam onto the selenium-coated drum

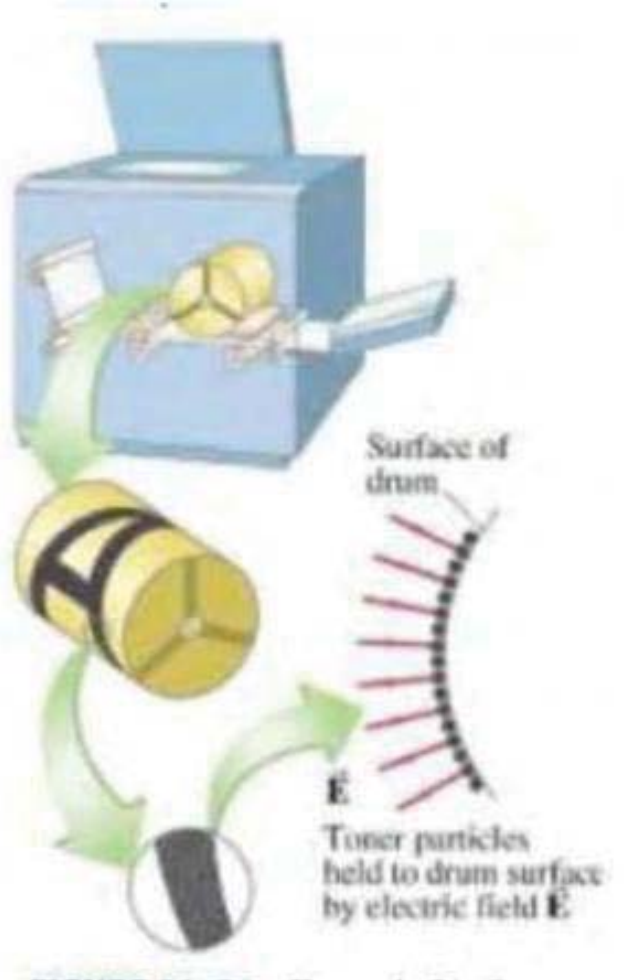


Example 13

- A photocopy machine works by arranging positive charges (in the pattern to be copied) on the surface of a drum, then gently sprinkling negatively charged dry toner (ink) particles onto the drum.
- The toner particles temporarily stick to the pattern on the drum and are later transferred to paper and "melted" to produce the copy.
- Suppose each toner particle has a mass of 9.0×10^{-16} kg and carries an average of 20 extra electrons to provide an electric charge.

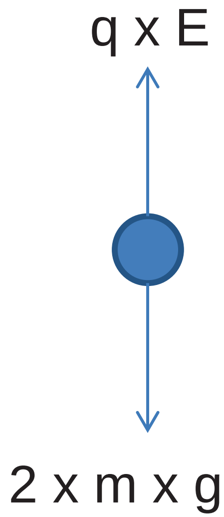


- Assuming that the electric force on a toner particle must exceed twice its weight in order to ensure sufficient attraction,
- compute the required <u>electric field</u> strength near the surface of the drum.



Answer :

- The electric force on a toner particle of charge q = 20e is $F = qE$, where E is the needed electric field.
- This force needs to be at least as great as twice the weight (mg) of the particle.
- $q \times E = 2\ m \times g$
- $E = 2\ m \times g / q = (2 \times 9 \times 10^{-16} \times 9.8) / (20 \times 1.6 \times 10^{-16})$
- $E = 5.5 \times 10^{3}$ N/C




Thanks,..
See you next week (ISA),...